1996

1996-1. The Eisenbud–Levin formula for the index of a vector field singularity "drives" a global topological invariant (mapping degree) into the local algebra of the singularity. What becomes of the other global invariants, such as characteristic classes and numbers, under a similar localization (both in the complex and, especially, in the real case)?

1996-2. Calculate the cohomology and fundamental groups of complements of strata of codimension 2 (and higher) in the space of immersed plane curves. *In the case of higher codimensions of strata the homotopy* (*and hence homology*) *groups probably are trivial. It is interesting to compare the results with those for analogous problems concerning the spaces of versal deformations of germs of maps* $(\mathbb{R},0) \to (\mathbb{R}^2,0)$ (*stabilization over the growing complexity of singularities*).

1996-3. Prove that the n-th symmetric power of $\mathbb{R}\mathrm{P}^2$ is $\mathbb{R}\mathrm{P}^{2n}$.

1996-4. Prove that the caustic is diffeomorphic to the Maxwell stratum for the singularity B_4 and transfer this result to higher singularities B_k (taking into account the symplectic or contact structures). *The symplectic version was constructed by F. Napolitano in* NAPOLITANO F. Duality between the generalized caustic and Maxwell stratum for the singularities B_{2k} and C_{2k}. *C. R. Acad. Sci. Paris, Sér.* I *Math.*, 1997, **325**(3), 313–317.

1996-5 (P. G. Grinevich)**.** Let $f(x)$ be a real Fourier integral,

$$f(x) = \int_{-\infty}^{\infty} F(k)e^{ikx}\, dk, \quad F(-k) = \overline{F(k)},$$

with vanishing low-frequency harmonics [$F(k) = 0$ for $|k| \leq \omega$]. Then the limiting averaged number of zeros of f on long intervals is not less than the averaged number of zeros of the function $\cos \omega x$ (i. e., the limiting density of zeros is not less than ω/π).

For a Fourier series the number of its sign changes on the circle is not less than the number of zeros for the lowest Fourier harmonic that has a non-zero coefficient in the series.

1996-6 (F. Aicardi)**.** Compare the following one-parameter families of hypersurfaces in the Euclidean space $\mathbb{R}^3$ given by a positive definite quadratic form f: a) the family of equidistants from the ellipsoid $f = 1$; b) the family of “quadraticoids” defined by the support functions $f + t$ on the unitary sphere.

Calculations show that in these families, when t varies, the perestroikas are topologically equivalent for corresponding (different) forms (and, moreover, the bifurcation diagrams in the spaces of the parameters defining the forms are diffeomorphic).

Explain this equivalence of families, by constructing the natural mapping between them. Does it hold in $\mathbb{R}^n$?

A quadraticoid and an equidistant, for two chosen corresponding forms, define the same fields of crosses on the Gauss sphere (*images under the Gauss map of the fields of principal directions*).

Question: *Is the entire set of perestroikas occurring in these families topologically necessary for the eversion of a front realized by the Legendrian collapse* (*or even by any Legendrian isotopy*)?

1996-7. Consider a typical function $z = f(x, y)$ of two variables. The asymptotic directions ($d^2 f = 0$) on its graph determine a field of crosses in the hyperbolic domain of negative Hessian determinant (with standard singularities on the boundary, which consists of the parabolic points where the Hessian determinant vanishes and the crosses degenerate into straight lines). What restriction on the topology of the field of crosses (i. e., on the section of the corresponding bundle over the hyperbolic set) are imposed by the hypothesis that this field arises from a function as the field of asymptotic directions?

1996-8. Investigate the multiplicities and the transversal multiplicities of the Lyashko–Looijenga mapping for polynomials, Laurent polynomials, modular polynomials on various strata and pairs of strata. *For polynomials the solution has been given by D. Zvonkine*; *the transversal multiplicities are the same in all cases.*

1996-9. M. Barner defines a *strongly convex* curve in $\mathbb{R}\mathrm{P}^n$ as a curve such that for every $n-1$ of its points there is a hyperplane passing through them and not intersecting the curve elsewhere. For example, a curve whose projection from a point to a hyperplane is convex in $\mathbb{R}\mathrm{P}^{n-1}$ is strongly convex in Barner’s sense in $\mathbb{R}\mathrm{P}^n$.

Investigate the manifold of strongly convex curves: the number of its connected components, singularities of the boundary, properties of dual curves, the existence of strongly convex projections and suspensions.

1996-10. Let us say that a plane of codimension 2 in the projective space $\mathbb{R}\mathrm{P}^{2n}$ is *interior* with respect to a convex curve if each hyperplane containing this plane intersects the curve at $2n$ points. Do there exist interior planes? What are the topological invariants of the manifold of such planes? *For $n = 1$, the interior planes are the points in the region bounded by the curve. The problem has been solved* (*affirmatively*) *by S. S. Anisov and S. M. Gusein-Zade.*

1996-11. Let us say that a straight line in the projective space $\mathbb{R}\mathrm{P}^{2n}$ is *exterior* with respect to a convex curve if, through every point of this line, $2n$ tangent hyperplanes pass. Do there exist exterior lines? What are the topological invariants of the manifold of such lines? *For $n = 1$, the exterior lines are those disjoint from the curve. The problem has been solved* (*affirmatively*) *by S. S. Anisov and S. M. Gusein-Zade.*

1996-12. Evaluate the cohomologies of the subgroups of the braid group corresponding to the coverings L2 (Lyashko–Looijenga) and L3 (Lyashko–Looijenga–Laurent) of the complement of the swallowtail.

The classifying $K(\pi, 1)$ spaces of these groups are known: *they are the complements of the bifurcation diagram of the spaces of ordinary and Laurent polynomials, respectively.*

1996-13. Investigate the variety of rational functions with three poles and the mapping L4 on it (taking a function to the set of its critical values).

1996-14. Define and explore the Morse complex of a solenoidal vector field in $\mathbb{S}^3$ (determined by the function on the space of closed curves whose value on a curve equals the field's flow through a surface bounded by this curve).

The extremals of this functional are closed trajectories of the field. The second differential has infinitely many both positive and negative squares, but one may try to examine "index difference" for a pair of closed trajectories with the help of bifurcation theory. If, moreover, the field is Legendrian with respect to some contact structure, then one may try to calculate such difference of indices of two closed trajectories using the geometry of the restriction of the contact 1-form to a surface whose boundary is the difference of these trajectories.

1996-15. Consider a discrete subgroup of the isometry group of the Lobachevskian plane [for example, the modular group $\mathrm{SL}(2, \mathbb{Z})$]. This group acts not only on the Lobachevskian plane but also in the de Sitter world (represented by the hyperboloid $x^2 + y^2 - z^2 = 1$ of one sheet in the Klein model, where the Lobachevskian plane is modeled by a sheet of the two-sheeted hyperboloid $x^2 + y^2 - z^2 = -1$).

To the metric of the Lobachevskian plane, there corresponds an invariant Lorentzian metric on the de Sitter hyperboloid. In the projective model, the

Lobachevskian plane corresponds to the interior of the unit disk, and the de Sitter world, to its exterior; in both cases, the geodesics are the straight lines and the isometries are the projective transformations of the plane that leave the separating circle invariant.

How is the dense orbit of a point in the de Sitter world under the action of the discrete group under consideration (e. g., of the modular group) distributed? Is it possible to define pseudo-fundamental domains, replacing the Voronoĭ polygonal domains on the Lobachevskian plane, for this world? *The question is provoked by works of E. Brieskorn and his successors on monodromy groups.*

1996-16 (Generalization of the Chevalley theorem?)**.** The Coxeter group $D(n)$ acts on the space $(\mathbb{C}\mathrm{P}^1)^n$ as follows: to a permutation of coordinates in $\mathbb{R}^n$, there corresponds a permutation of factors, and to the change of sign of a coordinate, there corresponds the antipodal involution of a factor. The manifold of orbits of this action is diffeomorphic to S^{2n} (this is the Maxwell–Sylvester theorem of the theory of spherical functions). We obtain a real linear action of a $(2n-1)$-dimensional Lie group in $\mathbb{C}^{2n}$ with smooth orbit manifold $\mathbb{R}^{2n+1}$. How can we describe all such actions?

1996-17. Consider a sign-changing generic smooth function F on the plane 2-torus. Study the motion of a charged particle with small energy in such a magnetic field (that is, the curves of geodesic curvature F/ε with $\varepsilon \to 0$ at each point).

In the region where $F \neq 0$*, the particle experiences a Larmor rotation along a circle of small radius* ε/F *the center of which slowly drifts along a level line of the function* F*. The trajectories intersecting the line* $F = 0$ *consist of loops with alternating orientation joined by segments of a trajectory whose inflection points lie on the curve* $F = 0$*. It is required to write the corresponding asymptotic formulae in a neighborhood of the curve* $F = 0$ (*where the assumptions of the standard averaging method are violated*) *and, in particular, evaluate the drift direction.*

Would these evaluations lead to counterexamples for the problem about four closed phase trajectories homotopic to a fiber of the sphericized (*co*)*tangent bundle of the torus in the case where the magnetic field F changes its sign*?

1996-18. Consider a generic positive smooth function F on the standard sphere S^2. Study the motion of a charged particle at velocity 1 in such a magnetic field (i. e., examine the curves of geodesic curvature F at every point). Do there exist (two?) closed trajectories whose phase curves are homotopic to a fiber of the sphericized (co)tangent bundle of the sphere?

Such trajectories exist if the function F is sufficiently large. Is it true that they always exist for a zero-divergence Legendrian vector field of the natural contact structure in ST^*S^2 without singular points? Our phase velocity field does have these properties, and, in addition, it is transversal to the field of planes in ST^*S^2 determined by the Riemannian connection. The situation seems to be similar to that in the conjecture of A. Weinstein, which was proved by C. Viterbo, and can be modeled with the use of fields on $\mathbb{S}^3$ instead of ST^*S^2.

1996-19. Study the asymptotic curves on cubic surfaces in $\mathbb{R}P^3$ (for example, on those close to the plane $\mathbb{R}P^2$). Is this dynamical system integrable or chaotic? What is the design of the first return function on a parabolic curve? (To each point on the parabolic curve, this function assigns the next point where the asymptotic line returns to the parabolic curve.)

The 27 (complex) lines on a cubic surface are asymptotic lines, so we can learn something by applying the theory of normal forms nearby.

1996-20 (M. B. Sevryuk)**.** Introduce the following definition. A symplectic structure is said to be *r-exact* if its r-th exterior power is exact whereas the $(r-1)$-th power is not ($r \in \mathbb{N}$). In particular, 1-exact structures are just exact ones.

Given a fixed number r, do systems Hamiltonian with respect to r-exact symplectic structures possess any special properties?

1996-21 (M. B. Sevryuk)**.** Does there exist a smooth vector field on $\mathbb{R}^n$ irreversible with respect to any phase space involution but such that its time 1 flow map is reversible?

If the answer to this question is affirmative then: Does there exist a smooth vector field V on $\mathbb{R}^n$ possessing the following properties: 1) the field V is irreversible with respect to any phase space involution, 2) for each $\tau_0 > 0$, there is $\tau \in (0;\tau_0)$ such that the time τ flow map of the field V is reversible?