1995

1995-1. Explore the topology of the stratification of the space of trigonometric (real and complex) polynomials modulo topological equivalence.

1995-2. Investigate mappings of Lyashko–Looijenga type for rational functions, especially in the cases of two poles (Laurent polynomials) and three poles ("modular polynomials"), when the set of poles has no moduli and the answer does not depend on the location of poles.

Evaluate the multiplicities of these mappings on various strata of the discriminant (generalizing Cayley's formula for the number of trees).

1995-3. Prove that a surface dual to a small perturbation of the projective plane in $\mathbb{R}P^3$ has at least four connected cuspidal edges (Aicardi's conjecture), even at the level of infinitesimal perturbations.

B. Segre proved that this is true for a cubic surface, and attempts to find counterexamples by the aid of higher order spherical harmonic functions were unsuccessful. The number of swallowtails on the dual surface is found to be not less than 6. *If the decomposition of the perturbation into spherical harmonic functions does not contain cubic harmonics and starts with fifth order harmonics then, according to Aicardi's examples, one obtains at least* 8 *connected cuspidal edges and at least* 14 *swallowtails.*

Counterexample: *D. Panov,* 1997 (*published in*: PANOV D.A. Parabolic curves and gradient mappings. *Proc. Steklov Inst. Math.*, 1998, **221**, 261–278): *there exist smooth perturbations of the projective plane in* $\mathbb{R}P^3$ *having only one parabolic line.*

1995-4. A point on a smooth plane curve is called an n-inflection point if the order of tangency with a suitable algebraic curve of degree n at this point is higher than usually. For example, the 1-inflection points are the ordinary inflection points (where the multiplicities of the intersections of the curve with its tangents are at least 3). The multiplicity of the intersection with the nearest curve of degree n usually equals $(n^2+3n)/2$.

How many 4-inflection points does a plane curve carry if it is sufficiently smoothly close to a) a circle, b) a cubic oval, c) an oval of a fourth-degree curve? Similar questions can be asked for any n.

Any convex curve carries at least six 2*-inflection points* (*the intersections at these points have multiplicity* 6; *for this reason, such points are called* sextactic). *A curve smoothly close to a circle has at least eight* 3*-inflection points* (*and there* *exist such curves with precisely eight points of nondegenerate* 3*-inflection*). *But a curve smoothly close to an oval of a cubic curve has not less than ten* 3*-inflection points* (*the intersections with suitable cubics are of multiplicity* 10 *at these points*). *It is interesting to determine where the boundary between the "closeness to an oval of a cubic" and the "closeness to a circle" passes, and what happens on this boundary. Possibly, when the higher derivatives are taken into account, the circle becomes an insufficiently convex curve, and there exists an interesting class of n-convex plane curves with specially good properties for each n.*

1995-5. The caustic of a general Lagrange collapse over $\mathbb{R}^3$ has at least three cusp edges (a conjecture of V. M. Zakalyukin). *Three edges are realized in an ellipsoid's caustic*; *thus, the conjecture asserts that the case of an ellipsoid is minimally complicated*: *the encountered singularities are topologically necessary.*

1995-6. Construct a parametric Morse theory that substantiates the topological necessity of the presence of complex critical points of functions on the fiber under certain parameter values, in terms of the topological complexity of the bundle on the total space of which the initial smooth function is defined. Carry over this theory to set-valued functions (that is, Lagrangian intersections).

1995-7. Study the singularities of the manifold of real projective curves completely decomposable into real lines.

The Maxwell–Sylvester theory of spherical harmonics asserts that this strange submanifold of the projective space of n-th degree curves is "linked" with the complementary projective space of curves containing the imaginary circle $x^2+y^2+z^2=0$ *as a component in a surprising way* (*namely, through each point of the complement of both spaces, there passes precisely one straight line joining them and intersecting each of them at one point*). *Do there occur other such "links"*?

1995-8. Find the simplest (i. e., with the minimal number of singularities) pairs of positively co-oriented curves immersed in the plane having equal Legendrian knots in $ST^*\mathbb{R}^2$, for which no regular homotopy without equally directed self-tangencies has been constructed (and try to prove that the latter does not exist).

1995-9. Find the simplest pairs of positively co-oriented curves (or fronts) immersed in the plane for which equipped knots coincide but Legendrian equivalence of knots in $ST^*\mathbb{R}^2$ has not been proved (and try to prove Legendrian nonequivalence).

1995-10. Find the simplest front with zero Maslov index whose Legendrian knot in $ST^*\mathbb{R}^2$ has not been realized by a Legendrian curve with smooth front (and try to prove that such a realization does not exist).

1995-11. How can we evaluate the minimum number of inflection points on realizations of a given class for immersions of the circle into the plane (sphere, projective plane, surface of genus g with the Lobachevskian metric)? *For example, the figure eight has not less than two inflection points on the plane, and it can have none on the sphere.*

There is a paper by B. Z. Shapiro on this topic; *cf. the dissertation of E. Ferrand* (*who proved the symplectic or contact equivalence of the family of curves of an Hadamard manifold to a standard one*; *in particular, for the Lobachevskian plane, this gives all four-vertex-type results*).

1995-12. Transfer the theory of completely integrable Hamiltonian systems from symplectic geometry to contact geometry (where, e. g., the Lagrangian invariant manifolds with their natural affine structures determined by Lagrangian fibrations must be substituted by Legendrian invariant manifolds with their natural projective structures determined by Legendrian fibrations). Carry over the Liouville theorem to this context and find applications to the infinite-dimensional case (where the equations of characteristics are partial differential).

1995-13. Is the "last geometric theorem" of Jacobi valid, according to which the first caustic (the set of first conjugate points to an arbitrary "pole" along all geodesics starting from it) of a typical ellipsoid has exactly four cusps?