1993

1993-1. Carry over the theory of neighborhoods of elliptic curves in holomorphic surfaces to pseudoholomorphic surfaces (develop theories of normal forms, resonances, bifurcations, series divergence, ...).

1993-2 (G. Moore)**.** Is there a relation between the invariants $J^{\pm}$, St of plane curves and polynomials in the areas of the components of curves' complements and their exponents arising in the theory of dual asymptotics of multiplicative integrals over Wilson loops (V. A. Kazakov, Yu. M. Makeenko, ...)? *Solved by M. B. Polyak in* 1997.

1993-3. Study the surface of changing four vertices for six in general families of curves $f_{a,b}(x,y) = c$ on the Euclidean plane with parameters a, b, c such that, at $a = b = 0$, the function f has critical point of minimum c with symmetric second differential $K(dx^2 + dy^2)$.

The supposed answer: a "dish" whose horizontal section has the form of a six-vertex hypocycloid and vertical sections through the axis are parabolic. But, more likely, there are functional moduli.

1993-4. Study vanishing flattenings in general families of curves in $\mathbb{C}^n$ given as preimages of a general mapping $f: \mathbb{C}^n \to \mathbb{C}^{n-1}$. Find the numbers of vanishing flattenings, bifurcation diagrams, and so on.

Certainly, the normal form of the mapping f does not give an answer by itself: the preimage should be subjected to a general diffeomorphism. For instance, at $n = 2$, the normal form $f = xy$ gives no planar points on the curve $xy = c \neq 0$. The correct (Plücker) answer —6 vanishing inflection points—is only given by the equivalent mapping $f = x^2 - y^2 - y^3$.

1993-5. Find all weight systems of multiplicative generators of a commutative $\mathbb{N}$-graded $\mathbb{C}$-algebra with the simplest Poincaré series $1+t+t^2+\cdots$, for which the classification of such algebras with respect to a) isomorphism of algebras, b) isomorphism of graded algebras is simple (has no modules).

For three generators, and any weights $1 < u < v$, the number of algebras is $2(a_1 + a_2 + \cdots) + 1$ where $v/u = a_0 + 1/a_1 + \cdots$ is a continued fraction. In the case of four generators, there are Sturmfels' examples of a nonsimple weight system, for instance, $(1,3,4,7)$, $(1,3,4,9)$, $(1,4,5,6)$. Unfortunately, the complete list of weights for which there are no modules is unknown even for 4 generators.

1993-6. Describe the Fintushel–Stern numbers related to the Floer numbers of quasihomogeneous knots of homology 3-spheres in terms of Newton polyhedra (admitting a multidimensional generalization).

According to FINTUSHEL R., STERN R. J. Integer graded instanton homology groups for homology three-spheres. *Topology*, 1992, **31**(3), 589–604*, the Poincaré polynomials of the Floer homology of the manifolds* $x^a + y^b + y^c = 0$, $|x|^2 + |y|^2 + |z|^2 = 1$ *have the form*

2 3 5	$t + t^5$	2 3 7	$t^{-1} + t^3$
2 3 11	$t + t^3 + t^5 + t^7$	2 3 13	$t^{-1} + t + t^3 + t^5$
2 3 17	$t + t^3 + 2t^5 + t^7 + t^9$	2 3 19	$2t^{-1} + t + 2t^3 + t^5$
2 3 23	$t + 2t^3 + 2t^5 + 2t^7 + t^{11}$	2 3 25	$2t^{-1} + 2t + 2t^3 + 2t^5$
2 3 29	$t + t^3 + 3t^5 + 2t^7 + 2t^9 + t^{11}$	2 3 31	$2t^{-1} + 2t + 3t^3 + 2t^5 + t^7$

1993-7. If the class of a plane curve (the orbit of a typical immersion $\mathbb{S}^1 \to \mathbb{R}^2$ under the action of the groups of orientation-preserving diffeomorphisms of $\mathbb{S}^1$ and $\mathbb{R}^2$) is symmetric (invariant) with respect to a symmetry (reflection of $\mathbb{S}^1$ or $\mathbb{R}^2$, or both), then this class has a representative which is a symmetric curve (instead of the diffeomorphisms, we can take the second-order isometries; they transform the immersion into itself).

A similar assertion is valid for atypical curves, that is, for various other orbits or strata of the manifold of immersions. But in different cases (*it seems, even for curves in* $\mathbb{R}^3$), *there occur symmetric classes without symmetric representatives. Is there a simple criterion for the existence of such symmetric representatives* (*in the classification problem for maps, immersions, embeddings, …*)?

1993-8. Vanishing Chern classes. In addition to vanishing inflections, we can consider other, non-point, strata of singularities on the dual hypersurface. They correspond to (singular) submanifolds of the initial hypersurface with various dimensions enumerated by the types of critical points of functions. The germs of these submanifolds at a singular point determine their infinitesimal analogues in the local ring of the singularity. The problem is to give an algebraic description (e. g., in the form of a flag or quiver of ideals in the local ring) and calculate the discrete invariants of the obtained algebraic objects for each singularity of the initial hypersurface.

1993-9. Can we join the curves  and  in the class of fronts of the Legendrian immersions in $ST^*\mathbb{R}^2$ having two (or having at most two) cusps?

1993-10. Consider two plane immersed curves in the same J^+ class. Join them by a generic path in the space of immersions, along which no perestroikas J^+

happen (i. e., no equally oriented self-tangencies). Consider the minimal number of the (other) perestroikas on such a path. Is this number bounded by a constant depending only on the complexity (the number n of double points) of the initial curves? If yes, how does this function grow with n? May be, it is not computable because of its growing faster than any computable function?

Is the problem of determining whether two curves belong to the same J^+-component algorithmically solvable (*probably, not*)?

Similar questions arise for all classification problems considered at the seminar; *for example, for* St-*classes, for fronts, for fronts with a fixed or an upper bounded number of cusps, etc.*

1993-11. Do the periodic continued fractions satisfy the Gauss statistics for the elements? For instance:

A) One can consider random matrices in $\mathrm{SL}(2,\mathbb{Z})$ or in $\mathrm{GL}(2,\mathbb{Z})$ in a large ball of radius R, expand them in continued fractions, and explore

a) the statistic of the elements of these periodic fractions;

b) the statistic of the period length.

Does the limit of the distribution as $R \to \infty$ exist, and does it coincide with the Gauss distribution? Is this limit the same for any homothetically widening domains in place of balls?

B) One can also consider random trinomials $\lambda^2 + a\lambda + b$ (with real roots) in the domain $a^2 + b^2 \leq R^2$ in $\mathbb{Z}^2$ and explore the statistics over these trinomials (one may also use other domains, e. g., $|a| \leq R$, $|b| \leq R$).

C) One can even start with rational fractions p/q, expand them in continued fractions, and try to calculate the limit of the statistics for $p^2 + q^2 \leq R^2$, $R \to \infty$; again, one may replace the disks with other domains. *Conjecturally, the answer is independent of the shape of the domains and, in all the cases, it is the same, as the Gauss invariant measure of the endomorphism* $x \mapsto 1/x - [1/x]$ *of the interval* $(0;1)$ *into itself indicates.*

1993-12. Describe the action of the braid group (and of its subgroups corresponding to various non-isolated singularities of fibers) on the homology of generic orbits, i. e., of the manifold T^*F_{n+1} (F_{n+1} is the space of full flags in $\mathbb{C}^{n+1}$), specified by the coadjoint representation $A_n = \mathrm{SL}(n+1,\mathbb{C})$:

manifold $\mathbb{C}^{n^2+2n}$ of $(n+1)$-matrices with trace 0 $\supset$ nonsingular fiber $\simeq T^*F_{n+1}$ = nonsingular orbit

mapping $\det(A - \lambda E)$ ↓   ↓ fibration

characteristic polynomials $\supset$ noncritical values = complement to the swallowtail

1993-13. Does there exist any planar not necessarily symplectic connection in the Milnor stratification at least for A_2? In other words, can one choose Dehn twists along a parallel and a meridian on a torus with hole V so that they satisfy the relation $aba = bab$ in the group $\mathrm{Diff}\,V$ [or better in the group $\mathrm{Diff}(V, \partial V)$ leaving all points of the boundary stationary], but not in $\pi_0(\mathrm{Diff}\,V)$?

1993-14. In COHEN P. -B., WOLFART J. Dessins de Grothendieck et variétés de Shimura. *C. R. Acad. Sci. Paris, Sér.* I *Math.*, 1992, **315**(10), 1025–1028, the Lobachevskiĭ triangles with angles $\frac{\pi}{p}, \frac{\pi}{q}, and \frac{\pi}{r}$ and the groups generated by these triangles are considered. We know that among these triangles, there are 14 especially remarkable ones (physicists' "mirror symmetry" = the "strange duality" between the Gabrielov–Dolgachev numbers). The question is, whether or not these 14 triangles are somehow distinguished in the arithmetic-topological theory of Galois–Grothendieck–Shabat, too.

1993-15. In a paper by Mourtada [MOURTADA F. -Z. Familles génériques à quatre paramètres de champs de vecteurs quadratiques dans le plan. Singularité à partie linéaire nulle. *C. R. Acad. Sci. Paris, Sér.* I *Math.*, 1993, **316**(7), 673–678] (presented by R. Thom to the PDE section for some reason), bifurcations of phase portraits are given, and the abstract of the paper claims that all portraits in domains contained in the complement of the bifurcation diagram are considered, while in the text (at the end), it is mentioned that the limit cycles are not studied. It is necessary to finish the study of limit cycles in the context of this paper (at least, determine their number!) and describe their bifurcations.

1993-16. Pierre-Louis Lions has recently been awarded a prize for a study of the influence of small viscosity on the Hamilton–Jacobi–Bellman equation; the prize announcement says that he invented viscous solutions and proved their convergence to shock waves in an appropriate sense.

As far as I remember, some work in this direction has been done before Lions (in particular, by S. N. Kruzhkov). How is this work related to Lions' results? What new contribution has Lions made?

1993-17. Is there the following fact in popular literature: The binomial coefficient C_i^x coincides modulo p^p (p is an odd prime) with the value of a degree x polynomial in i having *integer* coefficients if $x < p$?

1993-18. In *C. R. Acad. Sci. Paris, Sér.* I *Math.*, 1993, **316**(5), 513–518, a weird paper [FLIESS M., LÉVINE J., MARTIN PH., ROUCHON P. Défaut d'un système non linéaire et commande haute fréquence] about employing rapidly oscillating actions in control is published. The authors criticize the notions of complete controllability etc. and suggest something instead. This paper needs be thoroughly investigated, because the authors appeal to differential algebra, which by no means can be relevant. Have the authors obtained new results concerning the considered problems about inverted ordinary and double pendulums with rapidly oscillating suspension points?

1993-19. In *C. R. Acad. Sci. Paris, Sér.* I *Math.*, 1993, **316**(6), 573–577 there is the paper POLLACK R., ROY M.-F. On the number of cells defined by a set of polynomials, where for n variables and s equations of degree d in $\mathbb{R}^n$, the number of components of sets determined by s equations *or* inequalities for any sign choice is estimated: $\leq O\big((sd/n)^n\big)$. The only reference is WARREN H. E. Lower bounds for approximation by nonlinear manifolds. *Trans. Amer. Math. Soc.*, 1968, **133**, 167–178.

Does this result follow from Petrovskiĭ–Oleĭnik theory? What is known in the case of full intersection: how many components are there if no inequalities are present? Or—for the complement of the union of s hypersurfaces—what is a hypersurface of degree sd? What is the reason for $(sd/n)^n$ here? In standard inequalities for a hypersurface of degree $sd = D$, one may rather expect D^n/n. For example, if $d = 1$ and $n = 2$ then the number of domains $\sim s^2/2$ but not $s^2/4$; by integration, it seems, in $\mathbb{R}^n$ for $d = 1$ we get: roughly speaking, $s^n/n!$, more precisely, $e^n(s/n)^n + \cdots \gg s^n/n^n$ which contradicts the result of the paper. Maybe the authors mean O for n fixed? Why do they then take all of n^n?

1993-20. Is it possible to evaluate the Casson invariant of knots of singularities (at least, for the Brieskorn singularities $x^a + y^b + z^c$, whose associated knots are not homology spheres) [the definition can be found in the paper LESCOP C. Sur l'invariant de Casson–Walker: formule de chirurgie globale et généralisation aux variétés de dimension 3 fermées orientées. *C. R. Acad. Sci. Paris, Sér.* I *Math.*, 1992, **315**(4), 437–440] by analogy with the evaluations performed by R. Fintushel and R. Stern for homology spheres? Can we obtain the signature of the Milnor fiber?

1993-21. In DAX J.-P. Points singuliers normaux, points singuliers normaux simples et modèles d'élimination. *C. R. Acad. Sci. Paris, Sér.* I *Math.*, 1992, **315**(3), 315–319, a classification of maps $X \to Y$ taking $A \subset X$ inside $B \subset Y$ is given. What is this, mapping diagrams or quite a new problem?

1993-22. In PECKER D. Courbes gauches ayant beaucoup de points multiples réels. *C. R. Acad. Sci. Paris, Sér.* I *Math.*, 1992, **315**(5), 561–565, unicursal space curves with maximum number of double points are constructed; they all turn out to be *real*. Thus, in the problem about *space* curves, as opposed to plane curves, *everything* can be realized in a real domain (including any sets of double points and cusps?). Is there a general phenomenon, namely, that the singularities of mappings to multidimensional spaces can be "driven" into real domains (i. e., realized at $\mathbb{R}$-points for $\mathbb{R}$-mappings)?

1993-23. I. Ekeland et al. have recently proved that each *centrally symmetric* (quadratically) convex closed surface in $\mathbb{R}^n$ has an *elliptic* (probably, nonhyperbolic and non-Jordan?) closed geodesic [DELL'ANTONIO G., D'ONOFRIO B., EKELAND I. Les systèmes hamiltoniens convexes et pairs ne sont pas ergodiques en général. *C. R. Acad. Sci. Paris, Sér.* I *Math.*, 1992, **315**(13), 1413–1415].

Is there an example of a *nonsymmetric* surface without *elliptic* (in the same sense) closed geodesics? In particular, is it true that any closed surface close to a sphere has an *elliptic* closed geodesic? If a surface is close to a triaxial ellipsoid, then this, *seemingly*, follows from the Poincaré–Birkhoff theorem (but I have *not* verified whether this does indeed—points with negative eigenvalues also have positive indices).

What can we do when the surface is close to a sphere? Probably, we could perform averaging over great disks and again apply the Poincaré theorem—has anybody done this? It is convenient to define the metric by a function of the form $f \cdot$ the standard metric.

The question is, how does the center of the instantaneous great disk approximating the trajectory move in this averaged motion? Probably, there arises a Hamiltonian system on the sphere specified in terms of f, and the Hamiltonian function is related to the integrals of f over the great disks; what functions are obtained under such an integration?

1993-24. Study the "caustic–Maxwell stratum" duality.

1993-25. Jürgen Moser has recently found a new version of KAM-type theorems: Consider the complex torus $\mathbb{C}^n/(\Gamma \approx \mathbb{Z}^{2n})$ with, say, $n = 2$ and the foliation $\omega_1\, dz_1 + \omega_2\, dz_2 = 0$, generally nonresonant. The complex structure is perturbed into an *almost complex* one.

Question: What becomes of holomorphic foliations? The answer is as follows: For the directions of complete Lebesgue measure, they survive (in higher dimensions of leaves, which are left unstudied, the foliation is also quasiholomorphic, i. e., the tangent plane is invariant with respect to the almost complex structure $J : T_xM \hookleftarrow$, $J^2 = -E$; but the one-dimensional leaves are *complex* rather than only almost complex).

Question: What becomes of my theory of bifurcations of elliptic curves on complex surfaces when the surfaces are *almost complex*? This topic is quite extensive, because everything needs to be explored from the very beginning, including normal forms of normal bundles, neighborhoods with positive, zero, and negative self-intersection numbers, resonances, their realization, the Grauert theorem on negative neighborhoods, and so on.

1993-26. Study the singularities of the manifold of normal matrices.

There is yet another excellent unexplored manifold, the Taylor series of one-to-one mappings of the disk $|z| < 1$ to the plane ("the coefficient problem for univalent functions"): stratify the boundary and investigate the singularities of small codimensions in the space of series.

1993-27. Second, third, and succeeding braid groups: noncommutative resolvents. This is a very old problem, and it is time to clear it up.

Consider a general projection of a hypersurface Σ_0 in $\mathbb{C}^n$ onto the hyperplane $\mathbb{C}^{n-1}$ (germ at zero). The *discriminant* is a hypersurface Σ_1 in $\mathbb{C}^{n-1}$ over which the number of preimages is less than the degree of Σ_0 in 0. We obtain the chain of the discriminants $\Sigma_i \subset \mathbb{C}^{n-i}$ of the projections $p_i : \mathbb{C}^{n-i+1} \to \mathbb{C}^{n-i}$ and the chain of the groups $\Gamma_i = \pi_1(\mathbb{C}^{n-i+1} \setminus \Sigma_{i-1}, b_i)$ (near 0), where $b_i \in \mathbb{C}^{n-i+1}$. We have $\Gamma_i = F_i/R_i$, where F_i is the group generated by loops around Σ_{i-1} in the fiber $p_i^{-1}(p_i b_i)$ and R_i is the normal closure in F_i of the subgroup generated by the products $(A_\varphi f)f^{-1}$; here $f \in F_i$ and A_φ is the action on F_i of the groups F_{i+1} by the braids $(\varphi \in F_{i+1})$.

Clearly, the generators of Γ_{i+1} (denoted by φ above) correspond to relations in Γ_i. Moreover, the generators α of the group Γ_{i+2} correspond to relations from R_{i+1} in Γ_{i+1} and, therefore, to "relations between relations" in Γ_i.

Thus, the relations (elements of R_{i+1}) in Γ_{i+1} correspond to the generators (elements of F_{i+2}) of the group Γ_{i+2}. If $\alpha \in F_{i+2}$ is such a generator, then A_α acts as a braid on F_{i+1} and takes φ to $A_\alpha\varphi$; the element $\xi = (A_\alpha\varphi)\varphi^{-1} \in F_{i+1}$ acts now on F_i. We obviously have the following "Poincaré's $d^2 = 0$ lemma":

$$A_\xi f \equiv f \quad \forall \xi = (A_\alpha\varphi)\varphi^{-1} \quad \forall f \in F_i.$$

Question: To what degree is R_{i+1} less than the subgroup $\widehat{R}_{i+1}$ of the braid group acting on F_{i+1} that is defined by

$$\widehat{R}_{i+1} = \left\{\sigma \in \mathrm{Br}(F_{i+1}) \mid \forall \xi = (\sigma\varphi)\varphi^{-1} \in F_{i+1} \ \ \forall f \in F_i \ \ A_\xi f \equiv f\right\}?$$

This is a general question referring to any germ of a hypersurface at a point.

Now, let Σ_0 *be a swallowtail* in $\mathbb{C}^{\mu=n}$ (it can be obtained from $\Sigma_{-1} = \{x, \lambda \mid x^{\mu+1} + \lambda_1 x^{\mu-1} + \cdots + \lambda_\mu = 0\}$ by projecting along the x axis in $\mathbb{C}^{\mu+1}$). Then $\Gamma_0 = \mathbb{Z}$, $\Gamma_1 = \mathrm{Br}(\mu+1)$, and $\Gamma_2 = F_2/R_2 \equiv \widehat{R}_2$.

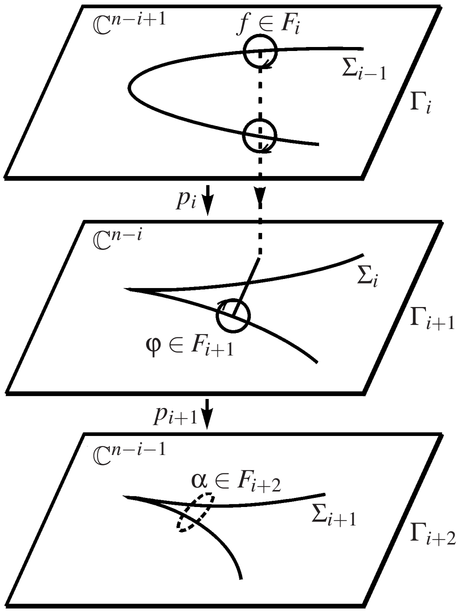

Fig. 1: The sequence of discriminants Σ_i and groups Γ_i

Urgent question: Is it true that $\Gamma_3 = F_3/R_3 \equiv \widehat{R}_3$? or $\widehat{R}_3 \supset R_3$? How can we describe Γ_3? Should we take quasihomogeneous, rather than general, projections p_i?

1993-28. Yet another old topic which it is timely to recall is singularities in "Cartan's geometric theory of PDE." The subject matter is systems of differential equations, that is, submanifolds in finite-order jet spaces, or, which is the same, modules of consequences.

1993-29. Suppose that v is a vector field in $\mathbb{R}^n$ which has a singular point and the real parts of the eigenvalues of its Jacobi matrix are negative (*everywhere*, rather than only at the given singular point). Is it true that the basin of attraction of this singular point is the entire space $\mathbb{R}^n$?

Perhaps, the condition will look less embarrassing if we consider the control system $\dot{x} = v(x) + u$; for an arbitrary u, the fixed point of this system is an attractor (with negative Lyapunov exponent).

1993-30. Compare the studies of the normal forms of Stokes surfaces performed by A. I. Neĭshtadt and S. K. Lando.

1993-31. M. R. Herman has presented a nice construction of an area-preserving diffeomorphism of a disk with positive Lyapunov exponents in the whole domain (see below). Is it possible to adapt this construction for solution of Sakharov's problem on fast ideal dynamo?

Recall that the collection of objects $\{A : B^3 \to B^3$ satisfying $\det A_* = 1$ and a divergence-free field v on the ball $B^3\}$ is called a *fast ideal dynamo* if $\iiint_B |A_*^n v|^2\, dx \geq C \exp \lambda n$, $\lambda > 0$.

The construction communicated by Herman: Let $A : \mathbb{T}^2 \to \mathbb{T}^2$ be an Anosov map, say, $\left(\begin{smallmatrix} 2 & 1 \\ 1 & 1 \end{smallmatrix}\right)$, and $\sigma : \mathbb{T}^2 \to \mathbb{T}^2$ be a holomorphic involution with 4 fixed points [e. g., the covering $w^2 = P_4(z)$ of an elliptic curve over S^2]. In $\mathbb{R}^2/\mathbb{Z}^2$, 4 points $(0,0)$, $(0,1/2)$, $(1/2,0)$, $(1/2,1/2)$ remain fixed under the action of A^6 (since $(0,0)$ is fixed under A, and the other points permute). Therefore, A^6 acts on the sphere (with 4 fixed points) as an Anosov system. Now it remains to resolve these 4 points.

1993-32. Multidimensional continued fractions and A-algebras.

D. Eisenbud has recently constructed an example (see below) of an A-algebra over $\mathbb{C}$ with moduli (not "simple"). Recall that an A-algebra is graded and has a Poincaré series $1 + t + t^2 + \cdots$ of polynomials in one variable. The degrees of multiplicative generators are determined uniquely: $1 = u_0 < u_1 < u_2 < \cdots$ (u_i fills the first lacuna in the degrees of monomials in lower-degree generators).

Thus, we can compose a Young diagram; for example, the anomaly $\alpha_i = u_i - i$ gives $1 = \alpha_0 \leq \alpha_1 \leq \alpha_2 \leq \cdots$. *Is there a relation* between the presence of moduli in A-algebras with given anomaly and in flattenings?

Eisenbud hopes to *prove* transversality at the Weierstrass points.

Certainly, most likely, there is no relation, but nevertheless, the simplicity (*the absence of moduli*) *selects simple ones among all Young diagrams, and of interest is the list of Young diagrams simple in this sense. The bifurcation diagrams also might deserve attention* (*though I do not know what they are, for the space of A-algebras is not linear*). *Maybe, we should consider one-dimensional extensions of the ready A_μ-algebra* (*of dimension μ over $\mathbb{C}$*), *because if an algebra has moduli, then they manifest themselves for the first time somewhere in the chain of extensions, and the space of extensions is less singular* (*it is not improbable that it is even not singular at all for one-dimensional extensions*).

Eisenbud's example: The generators are x_1, x_2, x_3, x_4, x_5, y_6, y_7, y_8, and x_{17} (the subscripts indicate their degrees); the relations are $x_i x_j = 0$, $x_i y_j y_k = 0$, and $x_1 y_i = x_2 y_i = 0$; the relations between y_i are the same as between y^i (e. g., $y_6 y_8 = y_7^2$; the multiplication by y_i acts on x_3, x_4, and x_5 as

$$x_3 y_7 = x_4 y_6, \qquad x_4 y_8 = x_5 y_7 = x_5 y_8 = 0, \qquad x_{17} y_i = 0,$$
$$x_3 y_8 = x_5 y_6, \qquad x_4 y_7 = a x_3 y_8 = a x_5 y_6.$$

We claim that a is a modulus. Indeed, multiplying the generator of degree i by λ_i for various i, we obtain that

$$\left.\begin{array}{ll} x_3y_7 = x_4y_6 & \text{implies } \lambda_3\lambda_7 = \lambda_4\lambda_6 \\ y_6y_8 = y_7^2 & \text{implies } \lambda_6\lambda_8 = \lambda_7^2 \end{array}\right\} \Longrightarrow (\lambda_4\lambda_7 = \lambda_3\lambda_8) \Longrightarrow a \text{ is a modulus!}$$

1993-33. Explore the asymptotic properties of random integer planes: Is their statistics similar to the Gauss statistics for continued fractions?

1993-34. Model the spectral sequence of a bundle by singularities in the same fashion as the homology complex is modeled by the Morse complex; namely, put geometric objects in correspondence with differentials and obtain "Morse inequalities," i. e., express the existence of some singularities (and bound characteristics of these singularities from below) in terms of differentials from the spectral sequence.

A concrete question: For the bundle

$$\begin{array}{c} S^{2n+1} \\ {\scriptstyle p}\big\downarrow{\scriptstyle \mathbb{S}^1} \\ \mathbb{C}\mathrm{P}^n \end{array}$$

and a generic function f on S^{2n+1}, specify the necessary multiplicity μ of a critical point of the restriction of f to a fiber $p^{-1}(b)$ for the worst fiber; the word "necessary" means "minimum over all generic f."

1993-35 (S. P. Novikov)**.** Consider a cyclic covering of a compact manifold and a general pseudoperiodic smooth Morse function f on the covering space (the differential of f is lifted from the initial compact manifold). Let $+1$ denote the action of the group $\mathbb{Z}$ on the covering space, and let $f(x+1) \equiv f(x)+1$. Suppose that f has critical points p and q of indices i and $i-1$, respectively. Consider the "instantons" (trajectories of the field grad f) joining the points p and $q-n$. Is the number of such instantons bounded by the exponent of n?

1993-36. Take a neighborhood U of a hyperbolic fixed point 0 of a diffeomorphism of the plane A. The *order* of a homoclinic point p (i. e., such that $A^m p \to 0$ as $m \to \pm\infty$) is the number of the points on the orbit of p that fall outside U:

$$\operatorname{ord}(p) := \#\{m \in \mathbb{Z} \mid A^m p \notin U\}.$$

Is the number of homoclinic points of given order n bounded by the exponent of n?

1993-37. A connected smooth hypersurface in the real projective space is said to be *locally hyperbolic* if its second quadratic form is everywhere nondegenerate. Is it true that all closed connected locally hyperbolic nonconvex surfaces in $\mathbb{R}P^3$ are quasiconvex and separate pairs of projective subspaces, having just two intersection points with every straight line connecting these subspaces (see problem *1990-4*?

1993-38. Is the set of closed connected locally hyperbolic nonconvex surfaces in $\mathbb{R}P^3$ connected? Is it true that any such surface has a convex plane section?

1993-39. Is it true that the generic caustic formed by the r-th conjugate points along the geodesics from a given point on the sphere S^2 has at least four cusps for any Riemannian metric on S^2?

1993-40. Is it true that the generic caustic formed by the r-th conjugate points along the geodesics from a given point on the sphere $\mathbb{S}^3$ has at least four D_4-type singularities for any Riemannian metric on $\mathbb{S}^3$?

1993-41. This problem and the six subsequent ones are devoted to critical points and Lagrangian singularities.

Let us consider a generic convex smooth closed curve γ on $\mathbb{R}^2$ and its normal lines. The unit vectors on these lines determining the same orientation as internal normals to γ form a Lagrangian submanifold M of the space $T^*\mathbb{R}^2$ (we identify tangent and cotangent vectors using the Euclidean metric of the plane). Whitney cusped singularities of the projection of this Lagrangian submanifold onto the plane—they are the curvature centers of γ for its vertices—are singular points of the caustic Γ consisting of the curvature centers of γ for all its points.

The manifold M is diffeomorphic to a cylinder. Unit vectors applied outside a large disk containing the caustic form two “collars” (semicylinders) on M. These collars are projected into the plane diffeomorphically, and the middle part of the cylinder—with singularities (the set of critical values is the caustic Γ).

Can the middle part of the cylinder M be replaced with another Lagrangian embedding, so that the resulting projection of the embedded Lagrangian cylinder into the plane has no Whitney cusped singularities (and coincides with the original projection on the collars)?

1993-42. A relaxed vesrion of the previous problem: can the middle part of the cylinder M be replaced with a Lagrangian *immersion*, so that the resulting projection of the *immersed* Lagrangian cylinder into the plane has no Whitney cusped singularities (and coincides with the original projection on the collar)?

1993-43. The cylinder M mentioned in problem *1993-41* is *optical*, i. e., it lies in the hypersurface $p^2 = 1$.

Can we replace this cylinder (the boundary collars being left intact) with an *optical* immersed (or embedded) Lagrangian cylinder whose projection on the plane has no Whitney cusped singularities?

1993-44. The topological invariants of the space of Morse functions on a given compact manifold (or of the space of functions whose critical points are not more complex than singularities from a given class) are interesting invariants of smooth manifolds; cf. ARNOLD V. I. Spaces of functions with moderate singularities. *Funct. Anal. Appl.*, 1989, **23**(3), 169–177; *the Russian original is reprinted in*: Vladimir Igorevich Arnold. Selecta–60. Moscow: PHASIS, 1997, 455–469.

Are the homotopy types of these function spaces determined by the topological type of the initial manifold, or do they indeed depend on the smooth structure?

1993-45. Consider a Morse function on a connected compact manifold. A suitable diffeomorphism sends all critical points of this function into a small ball in the manifold. The restriction of our function to a neighborhood of the boundary of this ball determines a Lagrangian (or Legendrian) collar, that is, the set of the first differentials of the function or its 1-jet at the points of a spherical annulus.

Is it possible to reconstruct a manifold from its Lagrangian collar? For what pairs of manifolds M_1^n and M_2^n do there exist functions $f_1 : M_1^n \to \mathbb{R}$ and $f_2 : M_2^n \to \mathbb{R}$ that coincide on balls containing all critical points?

1993-46. Consider a family of smooth functions as a function on the space of a smooth bundle (with compact base and fibers). Can the numbers of degenerate critical points (of different types) of the restrictions of this function to the fibers be estimated from below in terms of the topology of the bundle?

1993-47. Consider a smooth function in a neighborhood of a critical point 0 of finite multiplicity. Suppose that the index of the corresponding gradient vector field at 0 is zero. Consider the Lagrangian collar determined by the restriction of this function to a neighborhood of a sphere ∂B centered at 0. Does this collar bound a Lagrangian disk (or other Lagrangian manifold embedded in the cotangent bundle of the ball B) disjoint from the zero section?

1993-48 (M. B. Sevryuk)**.** Let a smooth involution $G : M \to M$ of an N-dimensional manifold M possess an invariant n-torus $L \subset M$, $L \cong \mathbb{T}^n$, the restriction $G|_L$ being conjugate to the transformation $\varphi \mapsto -\varphi$ (φ denotes the angular coordinate on $\mathbb{T}^n$) and therefore having 2^n isolated fixed points. What types of involution G can be at these points?

If $a \in M$ is a fixed point of the involution G then by definition the type of involution G at this point is $(p, N-p)$, whenever the linear part of G at the point a is a reflection in an $(N-p)$-dimensional plane. If

$$(p_1, N-p_1), \ldots, (p_{2^n}, N-p_{2^n})$$

are the types of involution G at fixed points $a_1, \ldots, a_{2^n}$ on the torus L, then $n \leq p_i \leq N$ for all i. Do all the collections of numbers p_i meeting these inequalities indeed occur?