

# 1992

**1992-1** (B. Teissier). Consider a function  $f$  in  $\mathbb{R}^n$  with a critical point of index zero. Is it possible to change  $f$  in an arbitrarily small neighborhood of this point so that the critical point disappears?

Suppose that the critical point of index zero has finite multiplicity. Is it true that there is a function in the class of versal deformations of  $f$  without critical points?

**1992-2.** Study the natural action of the braid group on the manifold of full flags and on the spaces of their cotangent bundles, which arises from the coadjoint representation of the group  $\mathrm{SL}(n, \mathbb{C})$ . Construct a monodromy and variation theory for non-isolated singularities which takes into account the tower of boundary conditions near strata of various dimensions intersecting the boundary of a ball centered at the point under consideration (instead of the condition that the monodromy is fixed on the boundary of the Milnor fiber).

*The obtained theory must apply to the mapping that assigns characteristic polynomials to matrices. It must generalize the Brieskorn–Grothendieck description of simple singularities for the  $A, D, E$  surfaces over nonquasiregular elements of Lie algebras (thus, the theory must apply to the family of four-dimensional intersections of general orbits with the local transversal to the corresponding non-general orbit).*

**1992-3.** Study the analytic continuation of elliptic curves embedded in the orbit space of a holomorphic mapping of a complex curve to itself. What singularities does the continuability boundary have? How large is this boundary (and the corresponding Riemannian surface)? *For instance, for the mappings  $z \mapsto z + \omega + \varepsilon \sin z$  and  $z \mapsto z + \omega + \varepsilon e^{iz}$ , where  $z \in \mathbb{C} \bmod 2\pi$ , we should consider continuation with respect to the parameters  $\omega$  and  $\varepsilon$  such that  $\varepsilon = 0$  and  $\mathrm{Im} \omega \neq 0$  [in this case, the curve is  $\mathbb{C}/(2\pi\mathbb{Z} + \omega\mathbb{Z})$ ].*

**1992-4.** Draw the discriminant of the family of odd polynomials  $x^7 + ax^5 + bx^3 + cx$ . Study the topological properties of this discriminant (such as the fundamental group, the number of components in the complement, stabilization, cohomology, etc.) in the  $\mathbb{R}$  and  $\mathbb{C}$  cases.

**1992-5.** Consider several elements  $a_1, \dots, a_k$  (e. g., with  $k = 2$ ) in the semigroup of germs of holomorphic mappings (or formal series) from  $(\mathbb{C}, 0)$  to itself [from

$(\mathbb{C}^N, 0)$  to itself]. Compose all words  $s$  of length  $n$  from the letters  $a_i$ . Suppose that every equation  $s(x) = x$  has a root of finite multiplicity  $\mu(s)$  at zero. Consider the maximum  $M(n)$  of this multiplicity over all (nontrivial) words  $s$  of length  $n$ . Can the function  $M(n)$  grow faster than an arbitrary prescribed function  $A(n)$  as  $n \rightarrow \infty$  (at least, on some subsequence of the  $n$  values) under a suitable choice of  $a_i$ 's?

*Or, maybe, we always have  $M(n) < Cn$  or  $M(n) < Ce^{\lambda n}$  with a constant  $C$  (depending on  $a_i$ ) for analytic  $a_i$ ? It is natural to ask similar questions about infinitesimal maps, that is, germs of vector fields at a point. It is then reasonable to compose words from multiple Poisson brackets of given fields (or even from their sums and differences) and estimate the order of zero in the field obtained.*

*All these problems, which are nontrivial even on the straight line ( $N = 1$ ), arise in studying bifurcations of limit cycles in relation to the Hilbert 16th problem, which is, in particular, concerned with estimating their number.*

**1992-6.** The Milnor fiber (say, of a simple singularity) has a natural symplectic structure (originating from the coadjoint representation). The vanishing cycles  $c_i$  are Lagrangian. Is it possible to represent the covanishing cycles ( $\text{var}^{-1} c_i \in H^*$ ) as Lagrangian submanifolds of the Milnor fiber such that their boundaries are Legendrian in the natural contact structure of the singularity knot? Can the intersection matrix (or even variation) of the Milnor fiber be described in terms of the obtained Legendrian link?

**1992-7.** Write explicitly an analytic (polynomial? trigonometric polynomial?) vector field without singular points in  $\mathbb{R}^5$  such that its smooth manifold of trajectories is homeomorphic but not diffeomorphic to the standard 4-space  $\mathbb{R}^4$ . Such a manifold is called a “fake” space  $\mathbb{R}^4$ .

**1992-8.** The double factorial appears as an answer in the classification problem of symplectic flags and in the list of Vassiliev diagram in knot theory. Is there a direct map, associating a symplectic flag to a diagram or a knot? *A natural symplectic structure on the space of knots (and even on the space of immersions with double points) does exist (J.-L. Brylinski): a curve representing a knot can be regarded as a point of a degenerate coadjoint orbit of the hydrodynamical group  $\text{SDiff}(\mathbb{R}^3)$ .*

**1992-9.** Does the Goresky–MacPherson theory of perverse sheaves have a distributed version where the constraints are imposed on ranks of chains with respect to the symplectic (contact) structure (or merely to the distribution) everywhere rather than only at the points of stratification?

*It is easy to invent a lot of definitions for, say, curves or surfaces in a symplectic or contact space (they are more likely to lead to cobordisms rather*

than to homologies, but we might try to set conditions at singular points too). The versions are so numerous that we need a rule for selecting suitable definitions.

**1992-10.** Calculate the moduli spaces of germs for hyper-Kähler structures; are their Poincaré series almost always (except for spaces of germs of infinite codimension) rational functions?

**1992-11.** Consider the Navier–Stokes equation (say, on the 2-torus) with external force proportional to the viscosity (Kolmogorov’s model). Is it true that, as the viscosity tends to 0 (i. e., the Reynolds number grows), there appear attractors of dimensions increasing with the Reynolds number (and containing no smaller attractors)?

Is it true that, moreover, the minimum dimension of all attractors unboundedly increases with the Reynolds number?

*A. N. Kolmogorov suggested (in 1958) that the answer to the first question was affirmative, but he doubted that so was the answer to the second because of the experiments on delaying loss of stability.*

**1992-12.** Prove exponential upper estimates (with probability 1) for topological invariants of the intersections  $(A^n X) \cap Y$  in the case where  $A$  is not a diffeomorphism, as in the note ARNOLD V. I. Dynamics of complexity of intersections. *Bol. Soc. Brasil. Mat. (N. S.)*, 1990, **21**(1), 1–10; *the Russian translation in:* Vladimir Igorevich Arnold. *Selecta–60*. Moscow: PHASIS, 1997, 489–499, but an arbitrary smooth mapping.

**1992-13.** Prove exponential upper estimates (with probability 1) for the number of periodic trajectories of period  $n$  of a typical diffeomorphism (or a smooth mapping of a manifold to itself).

**1992-14.** Does there exist, for any increasing sequence of positive integers  $a_n \rightarrow \infty$ , a polynomial vector field on  $\mathbb{R}^m$  (entering the spheres  $\{|\mathbf{x}| = r > 1\}$ ), which has more than  $a_{n_i}$  periodic orbits of periods  $\leq n_i$  for some increasing sequence  $n_i \rightarrow \infty$  (provided that all the periodic orbits are nondegenerate)?

**1992-15.** How large can the set of elliptic curves in the space of orbits of a polynomial vector field (or of a polynomial mapping, or of an algebraic correspondence with fixed discrete invariants, such as genera or degrees) be?