1991

1991-1. Consider the rotation field of a three-dimensional ball around an axis. Is it possible to decrease its energy to arbitrarily small values by acting on this field by volume-preserving diffeomorphisms? *Sakharov's conjecture* (1973): *it is possible for this field, but not for a field with at least one knotted trajectory or with at least one pair of linked trajectories.*

1991-2. The Bernoulli–Euler sequence (1, 1, 1, 2, 5, 16, 61, 272, 1385, ...) gives the numbers of topologically different Morsifications of the singularities A_n (i. e., the numbers of connected components in the complements of their bifurcation diagrams). What is the nonformal complexification of this theory? *The nonformal complexification of* π_0 *is* π_1. *Therefore, the answer is apparently the Lyashko–Looijenga covering.*

1991-3. Consider the recurrent sequence of degree n (say, 3)

$$x_{m+n} = a_1 x_{m+n-1} + \cdots + a_n x_m \quad (m = 0, 1, 2, \ldots).$$

Suppose that the number of zeros among the x_i is finite (the sequence is then said to be nonresonant). How many zeros can there be? Is their number bounded for a given n?

1991-4. Study the singularities of the manifold of normal operators.

1991-5. Study the singularities of the exponential mappings of Lie algebras (at least, of the matrix algebra) to groups (including the stratifications of singularity manifolds and uncovered parts of the groups, stabilization, local and global homotopy and homology groups of the complements of uncovered sets).

1991-6. Do the open umbrellas possess the Petrovskiĭ M-property (do the sums of Betti numbers of their complements in the real case equal those in the complex case)?

1991-7. Does the manifold of singular n-th degree polynomials in two variables possess the Petrovskiĭ M-property? *Singular = having less than* $(n-1)^2$ *different critical values.*

1991-8. Consider a linear operator $A : \mathbb{C}^m \hookleftarrow$ and two planes X and Y of complementary dimensions. Describe explicitly the conditions guaranteeing the existence of infinitely many integers n such that the space $(A^n X) \cap Y$ is of positive dimension.

1991-9. Construct a theory of connections with singularities. *Deform* (*in the sense of some equivalence*) *a given connection into a connection which is flat almost everywhere and its all the curvature is concentrated on a certain special submanifold. Then extract the invariants from the combinatorics of these singularities* (*and, possibly, from the "residues" of the connection at the singular points*).

1991-10. Is it true that a (smooth) pseudoperiodic curve in $\mathbb{R}^3$ has only one unbounded connected component? *Negatively solved by D. A. Panov.*

A pseudoperiodic curve is defined as the preimage of a point under a pseudoperiodic mapping $f : \mathbb{R}^3 \to \mathbb{R}^2$, *where* $f = (\textit{linear}) + (\mathbb{Z}^3\textit{-periodic})$ *and the incommensurability conditions* $\ker(\textit{linear}) = \mathbb{R}$, $\ker(\textit{linear}) \cap \mathbb{Z}^3 = \{0\}$ *hold* (*they almost always hold*).

1991-11. Consider the convex hull of the set of integer points in the pyramid $z > ax + by$, $x > 0$, $y > 0$ (a, b are arbitrary positive numbers). Examine the asymptotics of the polyhedral surface bounding this convex hull (for example, the mean number of edges in a vertex or on a face, the mean number of integer points on an edge; the probability that a random face is a triangle, a quadrangle, ...).

Generalize Gaussian distribution of continued fraction elements by transferring it to trihedral (*general*?) *pyramids in the space* $\mathbb{R}^3$ *containing* $\mathbb{Z}^3$. *In this situation, prove the multidimensional generalization of Lagrange's theorem on the periodicity of continued fractions*: *topological periodicity is present if and only if planes are eigenplanes of a lattice-preserving operator. The two-dimensional case shows that the boundary of the convex hull should be colored* (*where colors correspond to affine* $\mathrm{SL}(2,\mathbb{Z})$*-types of stars of vertices or generalized r-stars containing vertices connected to a given one by a path of at most r edges*). *In the two-dimensional case,* 1*-stars determine integer-valued angles of the boundary polygonal line*; *these, together with integer-valued edge lengths, are the elements of the continued fraction. The generalization of Lagrange's theorem to dimension* 3 *states that the topological periodicity of the coloring implies the pyramid's provenance from the eigenplanes of an* $\mathrm{SL}(3,\mathbb{Z})$ *operator.*

1991-12. Consider bundles whose fibers are surfaces, namely, the Milnor bundle for the A_n singularities of a function in two variables or the tautological bundle over the moduli space of curves of given topological type. The fundamental group of the base is represented by automorphisms of homology groups of the fiber (by means of the monodromy). Can it be represented directly into the group of diffeomorphisms (rather than of their isotopy classes)? A similar question can be asked for higher dimensions and symplectomorphisms.

In the case of A_1 and symplectomorphisms, the answer is affirmative for all dimensions: *there are the symplectic Dehn twists. In the case of A_2 and curves, there also exists an explicit construction. According to V. V. Fock, there is no representation into the homeomorphisms of a fiber for the $A_{\geq 4}$ curve singularities.*

1991-13. Examine the topological properties of the manifold of the Legendrian curves (immersed or embedded) disjoint from a given Legendrian knot (find its fundamental group and cohomology).

According to A. B. Givental, the space of all Legendrian submanifolds is similar to a Lagrangian Grassmannian, and the submanifold of those intersecting a given submanifold is similar to the trail of a Lagrangian plane (*formed by the Lagrangian planes intersecting it nontransversally*).

1991-14 (S. P. Novikov)**.** A submanifold of the Euclidean space $\mathbb{R}^n$ is called $\mathbb{Z}^r$*-periodic* if it is invariant under translations by the vectors of some integral sublattice $\mathbb{Z}^r \subset \mathbb{R}^n$. Consider a generic irrational (affine) planar section of a $\mathbb{Z}^3$-periodic surface (*Fermi-surface*) in $\mathbb{R}^3$. Is it true then that every unbounded component of this curve lies in the R-neighborhood (with a finite $R > 0$) of some straight line?