1989

1989-1. Classify the simple singularities of functions on supermanifolds.

1989-2. Can the number of fixed points of the n-th iteration of an infinitely smooth mapping of a compact manifold to itself grow, as n increases, faster than any prescribed sequence a_n (for some subsequence of time values n_i)?

1989-3. Calculate π_2(the complement of the stratum A_3 of the swallowtail in $\mathbb{R}^n$) for non-stable dimensions n.

1989-4. Study the cohomology rings of the complements of bifurcation diagrams of functions A_k in $\mathbb{C}^{k-1}$ (including the stabilization as $k \to \infty$, the behavior under the Lyashko–Looijenga mapping, and the relation to stratum diagrams). *This is the cohomology of the "second braid group," because the complement of a bifurcation diagram in $\mathbb{C}^{k-1}$ is $K(\pi, 1)$.*

1989-5. What functions on manifolds can serve as Jacobians?

1989-6. Give a relative version of the Moser theorem on symplectic structures (fix a submanifold and a 2-form on it).

1989-7. Carry over the inequalities of Harnack, of Petrovskiĭ, etc. to the pseudoperiodic hypersurfaces determined by sums of (incommensurable) harmonics of the form $A\cos((k,x)+a)$ in $\mathbb{R}^n$ (study the densities of topological objects in unit volumes). *For instance, we can divide by R^n the number of maxima, or the Betti numbers, or the Euler characteristic of the domain $f \leq c$ in a large ball of radius R and send R to infinity*; *it is required to estimate the limit "density of maxima," or "density of Betti numbers," or "density of the Euler characteristic" from above in terms of the number of harmonics* (*or, if possible, of the Newton polyhedron*).

1989-8. Nonconvex Minkowski problem. Given a generic mapping $S^2 \to S^2$ of degree 1, consider its Jacobian as a (set-valued) function on the image sphere. What conditions on this function ensure the existence of a Gauss mapping (of a sphere immersed in $\mathbb{R}^3$) with such a Jacobian?

In the absence of singularities, the only condition is that the center of gravity of the corresponding mass distribution on the sphere should be at zero (*the Minkowski theorem*).

1989-9. Classify the flags in a symplectic space and simple symplectic quivers.

1989-10. Study the systems of fronts and of rays defined by hyperbolic variational principles near typical singularities of the surface of zeros of the symbol (for two- and three-dinensional physical spaces).

1989-11. Classify the neighborhoods of Riemann curves of genus g on complex surfaces. *The case of an elliptic curve, $g = 1$, is studied in detail, e. g., in the following book*: ARNOLD V. I. Geometrical Methods in the Theory of Ordinary Differential Equations, 2nd edition. New York: Springer, 1988 (Grundlehren der Mathematischen Wissenschaften, 250); *the Russian original* 1978.

1989-12. The infinitesimal version of the problem about periodic orbits of correspondences: Let $A : \mathbb{S}^1 \to \mathbb{S}^1$ be a diffeomorphism of a real oval for an algebraic curve such that its analytic continuation is a correspondence on a Riemannian surface and $A^k = \mathrm{id}$. How many periodic orbits (of period n) can arise under a small perturbation of this diffeomorphism (in the class of real algebraic self-correspondences of the same bigenus and bidegree)? Is this number bounded by a function of n of by a constant independent of n (uniformly over perturbations or at least in the first approximation of perturbation theory)?

1989-13. In the problem of bypassing an obstacle, examine the asymptotics as the obstacle diffuses and turns into a steep potential.

1989-14. In the space of polynomials $\mathbb{R}^n = \{x^{n+1} + a_1x^{n-1} + \cdots + a_n\}$, consider the subvariety A_3 (of codimension 2) consisting of the polynomials with threefold roots. The fundamental group of the complement of this subvariety is $\mathbb{Z}$. The polynomial in two variables $x^{n+1} + a_1(y)x^{n-1} + \cdots + a_n(y)$ naturally defines a curve in $\mathbb{R}^n$. A generic curve does not intersect the subvariety A_3. Fixing the boundary conditions for $y \to \pm\infty$, we can associate with such a curve an integer [an element of $\pi_1(\mathbb{R}^n \setminus A_3) \approx \mathbb{Z}$] called the *index* and counting the number of rotations of the curve around A_3.

Find the minimal degree of the polynomial in two variables (or of polynomials a_j in y) for which a given value i of this index is realized.

Investigation of this question led V. A. Vassiliev to the problem on the minimal degree of a polynomial mapping $\mathbb{R} \to \mathbb{R}^3$ realizing a fixed knot. The investigation of the arising knot invariant led him to the theory of invariants of finite order.

1989-15. What is the maximum number of parts into which the sphere can be divided by the zeros of a spherical function being a polynomial of degree n?

The well-known Courant theorem gives the upper bound of $n^2/2 + O(n)$ (*for the 2-sphere*), *and examples of V. N. Karpushkin give the lower bound of* $n^2/4 + O(n)$.

What is the largest number of maxima for such a function?

1989-16. Find the number of components in the space of nondegenerate homogeneous equations $\dot{x} = P(x)$, where $x \in \mathbb{R}^n$ and the components of P are second-degree homogeneous polynomials having no common zeros but the origin.

The geometric problem (*for* $n = 4$) *reduces to studying deformations of quadruples of quadrics* (*ellipsoids*) *in the projective space. The quadrics are allowed to degenerate and even vanish, but they are forbidden to have a point common to all of them. The question is, how many quadruples are there that cannot be so deformed into each other?* (*For* $n = 3$, *triples of ellipses should be considered*; *in this case, the answer is* 2: *the ellipses from one triple are disjoint, and in the other triple, each ellipse separates the two intersection points of the two other ellipses.*)

1989-17. How many limit cycles can arise under a small polynomial (of degree n) perturbation of an integrable polynomial system of degree n?

The question reduces to exploring the number of zeros of the integral

$$I(h) = \oint \frac{P\,dx + Q\,dy}{M}$$

along ovals $H = h$ *of the system* $\dot{x} = X(x,y)$, $\dot{y} = Y(x,y)$ *with integrating factor* M, *where* X, Y, P, Q *are polynomials of degree* n. *It is unsolved even for* $n = 2$ *and even in the case* $M = 1$ *where* H *is a polynomial. In the case where* $M = 1$ *and* H, P, Q *are polynomials of a fixed degree, there is a uniform upper bound for the number of zeros* (*A. N. Varchenko, A. G. Khovanskiĭ*) *but it is ineffective.*

1989-18. The sequence of meandric numbers 1, 1, 2, 3, 8, 14, 42, 81, ... is defined as follows. Suppose an infinite river running from south-west to north-east intersects an infinite straight road going from the west to the east under n bridges numbered $1, \dots, n$ in the order from west to east. The order of the bridges along the river determines a *meandric permutation* of the numbers $1,\ldots,n$. The *meandric number* M_n is the number of meandric permutations on n elements.

Meandric numbers possess many remarkable properties; *for example*, M_n *is odd iff* n *is a power of* 2 (*S. K. Lando*). Find the asymptotics of M_n as $n \to \infty$. *It is known that* $c\,4^n < M_n < C\,16^n$ *for some constants* c, C.

1989-19. Is it true that the minimum Hausdorff dimension of a minimal attractor of the Navier–Stokes equation (on the 2-torus, say) increases with the Reynolds number?

Even the existence of some minimal attractors of dimensions growing with the Reynolds number is not proved; *only upper estimates for the dimensions of all attractors by powers of the Reynolds number* (*obtained by Yu. S. Il′yashenko, M. I. Vishik, and A. V. Babin*) *are known.*

1989-20 (V. P. Kostov)**.** Describe the singularities of the pseudo-Stokes hypersurface of a typical family of polynomials. *The pseudo-Stokes hypersurface of the family of polynomials* $x^n + a_1 x^{n-2} + \cdots + a_{n-1}$ $(x,\, a_i \in \mathbb{C})$ *is the set of values of the coefficients* a_i *for which two of the roots have the same real part.*