1987

1987-1. Carry over the theory of the Gibbs distribution (for the one-dimensional evolution $u_t + (uv)_x = \varepsilon u_{xx}$) to the case of a discrete time (a map $\mathbb{S}^1 \to \mathbb{S}^1$, close to the identity, is being perturbed by a small diffusion). What is an analog of the theory of eigenvalues close to zero that correspond to the point attractors of the field v?

1987-2. Symplectize the nonoscillation theory (including the Pólya theorem concerning factorization on an interval).

1987-3. The transformation $z \mapsto z^2$ sends the trajectories of small oscillations to the Newton ellipses. And what about the transformation $z \mapsto z^{\alpha}$?

1987-4. Consider hypersurfaces in $\mathbb{R}\mathrm{P}^n$ of constant signature, e. g., of signature $(1,1)$ in $\mathbb{R}\mathrm{P}^3$ (a compactification of the Hilbert problem on embedding a surface of everywhere negative curvature into $\mathbb{R}^3$).

a) Is it true that the space of such hypersurfaces is connected?

b) Is it true that any such surface separates two lines (in the case of a hypersurface of signature (k,l), separates $\mathbb{R}\mathrm{P}^k$ and $\mathbb{R}\mathrm{P}^l$ in $\mathbb{R}\mathrm{P}^{k+l+1}$)? *The answer is positive for $k = 0$: any convex hypersurface is affine.*

c) Is it true that any such hypersurface is a two-fold covering of $\mathbb{R}\mathrm{P}^k \times \mathbb{R}\mathrm{P}^l$ (or, better, that the subspaces $\mathbb{R}\mathrm{P}^k$ and $\mathbb{R}\mathrm{P}^l$ separated by the hypersurface can be chosen in such a way that any line joining them intersects the hypersurface at exactly two points)? *This is true for $k = 0$: a convex hypersurface is star-like with respect to any point of the domain bounded by this hypersurface.*

d) The "maximum principle": consider a hyperbolic surface lying in the strip $|z| \leq 1$ in $\mathbb{R}^3$ and tangent to the cone $x^2 + y^2 = z^2$ along the circles $z = \pm 1$. Prove that the surface does not intersect the interior $z^2 > x^2 + y^2$ of the cone. Generalize to other boundary conditions.

1987-5. Examine the global topological properties of the caustics and fronts of Legendrian manifolds (and, separately, of optical manifolds; their properties may be different!).

1987-6. Evaluate $\pi_3(\mathbb{C}^n \setminus \Sigma^{n-2})$, where Σ^{n-2} is the cuspidal edge of the swallowtail. *Of course, here also the similar questions in $\mathbb{R}^n$ and for strata of greater codimension and higher π_i are assumed.*

1987-7. How many connected components does the complement of the trail of a complete flag in a neighborhood of this flag in $\mathbb{R}^n$ have? *There are two for* $n = 2$ *and six for* $n = 3$.

1987-8. How many connected components are possessed by the complements of (i) bifurcation diagrams of functions and (ii) discriminants of (at least) simple singularities in spaces of real versal deformations?

1987-9. Is M. E. Kazarian's list of the Young diagrams of simple singularities a solution to some other classification problem?

1987-10. How does the number of critical points of the N-th eigenfunction of the Laplacian in an n-dimensional domain increase as $N \to \infty$? Like $N^{1/n}$?

1987-11. What singularities are encountered in solutions of the variational problem to minimize the Dirichlet integral $\int (\nabla u)^2 \, dx$ over all functions u obtained from a given one by the action of area-preserving diffeomorphisms of the domain (say, of the disk)? *If a given function vanishes on the boundary of the disk and has one maximum inside, then the extremum in the above problem is a centrally-symmetric function on the disk such that the area of the set where the latter function is less than a number equals the area of the set where the former given function is less than the same number. If the initial function has two maxima, similar to the two summits of Elbrus, and a saddle, then physicists observed in numerical experiments that the extremal function has singularities along the segment replacing the saddle.*

1987-12. Study the decomposition of the space of linear complex equations with singularities into isomonodromy classes (of special interest are the limits of isomonodromic systems with merged singular points, namely, their versal deformations, bifurcation diagrams, etc.).

1987-13. Study the degeneracies of symplectic structures in the space of closed 2-forms, namely, the stratification of the boundary of the manifold of symplectic structures, the bifurcation diagrams at the points of finite-codimensional strata on the boundary, ...

1987-14. Do there exist smooth hypersurfaces in $\mathbb{R}^n$ (other than the quadrics in odd-dimensional spaces), for which the volume of the segment cut by any hyperplane from the body bounded by them is an algebraic function of the hyperplane?

For these quadrics the volume is an algebraic function (*Archimedes*)*, and the area of segments of plane curves is never algebraic* (*Newton*)*.*

1987-15. Define the "asymptotic Sturm invariant" describing the mean Lagrangian oscillations of variational equations for a Hamiltonian system (in the same sense as the asymptotic Hopf invariant counts the mean number of zeros (with signs) for solutions to normal variational equations; the latter assertion also needs be formalized).

1987-16. Study the boundary of the set of second order linear equations with alternating roots of solutions (and carry over the results to the Lagrangian alternation in Hamiltonian systems with n degrees of freedom). Roots alternation property of a second order equation means: in the interval between any two roots of any solution there exists a root of any other solution.

1987-17. Are the transformations of phase flows of contact fields in $\mathbb{S}^3$ dense among the contactomorphisms from the identity component?