1986

1986-1. Consider the space of Lagrangian tori in $T^*\mathbb{T}^2$ that are isotopic to the zero section among all the tori. How many connected components does it have?

1986-2. Consider a Hamiltonian in $T^*\mathbb{T}^2$ quadratically convex with respect to momenta. Suppose that the tori mentioned in the preceding problem lie on its level-1 hypersurface (deformations are then applied to pairs torus–Hamiltonian). How many connected components are there in the space of such pairs (topologically trivial)?

1986-3. A rigid body is controlled by a momentum of a given intensity; the orientation of the momentum with respect to the body (satellite) can be taken as a controlling parameter. It is required to turn the body from one state to another (perform a rotation in $\mathrm{SO}(3)$) as fast as possible, say, at zero initial and final angular velocity.

Describe the optimal control (first and foremost, the topology of the manifold of the discontinuity of this control on $\mathrm{SO}(3)$).

1986-4. To a purely imaginary pair of a vectorfield's eigenvalues there corresponds, generally speaking, a Lyapunov invariant surface. Explore the perestroikas (bifurcations) of these surfaces at resonances.

1986-5. Transfer the Smale–Hirsch theory to the Lagrangian and Legendrian bands (germs of Lagrangian and Legendrian manifolds along curves belonging to these manifolds) or to the corresponding framed curves.

1986-6. Is the diameter of the symplectomorphism group of the ball B^{2n} bounded? Conjecturally, no. (*In the two-dimensional case this was proved by A. I. Schnirelmann. In the higher-dimensional case, thanks to non-simple-connectedness of the group of symplectic matrices* ($\pi_1 = \mathbb{Z}$), *one can strongly twist a central ball, and the corresponding diffeomorphism is conjecturally rather far from the identity*.)

1986-7. Find the asymptotic form of the number of meanders with $n \to \infty$ bridges.

1986-8. Study the singularities of the apparent contours of convex bodies.

1986-9. In optimization theory, there occur situations where a nonconstant (say, periodic) control gives better (on average over a long time) results than any fixed parameter.

Study these situations from the viewpoint of genericity and bifurcations. *The situation resembles a phase transition. Generally, the regime optimal on the average may be more complex than a periodic one*!

1986-10. Reformulate the theorem about three inflection points of a projective curve and about four vertices of a Euclidean curve in terms of symplectic or contact topology.

1986-11. In addition to models with internal degrees of freedom along a small fiber of a bundle over space-time, models of the surface tension type are conceivable, where the fundamental laws of hydrodynamics act in a larger space but an observer on the surface only sees their manifestations in a smaller space (the difference in the dimensions can even be greater than 1). What common features do the models of this type have—what is the structure of their equations of motion?

1986-12. Study the singularities of the level $u = 0$ for a function u of two variables satisfying the (Euler stationary) equation: there exists a function f such that $\Delta u = f(u)$. Investigate the typical cases and bifurcations of codimensions 1 and 2.