

1984

1984-1. Examine the singularities of the boundary of the space of Chebyshev systems of functions.

1984-2. Construct a Morse theory with nonholonomic constraints, say, for higher derivatives.

1984-3. Investigate global topological restrictions on caustics implied by the condition that the eiconal is positive definite.

1984-4. Prove that on \mathbb{T}^2 there are (generically) at least four closed (on the universal covering) curves of constant geodesic curvature $K > 0$.

1984-5. Consider the circle $x^2 + y^2 = 1$ and a quadratic function with parabolic level lines intersecting the circle not more than twice [e.g., $y + (x - a)^2$, $|a| > a_0$, where the value $a_0 > 0$ is determined by the condition that the parabola $y + (x - a_0)^2 = c_0$ has a point of *cubic* tangency with the circle under a suitable choice of c_0]. Consider the correspondence permuting the intersection points. The product of two such correspondences changing orientation (the second, for example, changes the sign of y or of x) determines an orientation-preserving diffeomorphism of the circle onto itself. Is the number of cycles (periodic trajectories) of this diffeomorphism bounded by a constant independent of a ?

1984-6. Classify the germs of “generic” Poisson structures in \mathbb{R}^3 . *The term “generic” needs to be defined. The situation is the same as in classifying Lie algebras or commuting pairs of functions on the symplectic plane and in similar problems: the initial infinite-dimensional space is not smooth and, generally, may have components of “different dimensions.”*

1984-7. Build the theory of versal deformations of Fuchsian systems. Is it true that regular singularities are isomonodromic limits of (confluent) Fuchsian points? Which matrices from the monodromy group converge to the Stokes matrices in the irregular case?

1984-8. Give an axiomatic definition of skew-symmetric versions of the monodromy groups of simple singularities (which would lead to their classification,

similar to the classification of reflection groups or Weyl groups in the symmetric case). Apply this definition to complete intersections (considering a flag of embedded hypersurfaces and sequences of root systems).

1984-9. Is the number of Dynkin diagrams (of strongly distinguished bases) of a fixed singularity finite?

1984-10. Describe variational and symplectic properties of Picard–Fuchs equations (the Gauss–Manin connection). Are they not the Euler equations for an appropriate group?

1984-11. Translate the relative Morse theory into the symplectic language of the theory of Lagrangian intersections or Legendrian links.

1984-12. Carry over the asymptotic ergodic definition of the Hopf invariant of a divergence-free vector field to S. P. Novikov’s theory generalizing the Whitehead product in homotopy groups.

1984-13. Does there exist a mapping $\mathbb{R}P^2 \rightarrow \mathbb{R}^2$ with only one Whitney cusped singularity? *Yes; solved by Yu. V. Chekanov on October 23, 1984.*

1984-14. What is known about \mathbb{C} -contact structures in \mathbb{C}^3 ?

1984-15. How can we extract information independent of the choice of generating loops in the successive fundamental groups of complements of points on the \mathbb{C} fibers of successive bundles from the “resolutions” of the fundamental groups of complements of algebraic hypersurfaces?

1984-16. Study the equation $dy/dx = f(x,y)$ where x and y are angular coordinates on the circle while f is a trigonometric polynomial: How many limit cycles can occur for a given Newton polygon?

1984-17. Prove that the standard symplectic space \mathbb{R}^4 contains no exact embedded Lagrangian torus.

1984-18. Complexify the Rolle theorem: if the image of the boundary of a disk equals 0 modulo 2, then the disk contains a critical point inside.

1984-19. Classify the umbrellas in a contact space (that is, germs at the vertex up to contactomorphisms).

1984-20. Calculate the number of vanishing inflections (of type A_n) at a singular point of a hypersurface A_2 in \mathbb{C}^3 (in \mathbb{C}^n) subjected to a generic diffeomorphism (if $n = 2$ then there are 8 inflection points of type A_2 —Plücker’s formula).

1984-21. Consider a “generalized Bernoulli scheme”—a network of identical automata with finite radius of action (and memory) in \mathbb{Z}^n ($n = 1$?). Can one derive from their work a difference approximation to something non-Gaussian (i. e., not to the equation of heat conductivity)? Just to what?

1984-22. Does there exist a finite number (as R. Thom assumed) of various (germs of) bifurcations of the phase portrait of a gradient system, generically depending on 4 parameters? *R. Thom stated that there are 7 types of such systems. According to B. A. Khesin, there are at least 13 of them, but probably their number is infinite?*

1984-23. Develop a “supertheory” whose even component corresponds to reversible systems, and odd one to Hamiltonian systems.