1983

1983-1. How many points (curves, ...) of inflections of various types are merged at a singular point of a hypersurface (subjected to a generic diffeomorphism)? *J. Plücker*: 6 *inflections meet in* A_1, *and* 8 *in* A_2.

1983-2. Courant's theorem says that the zeros of the n-th eigenfunction of the Dirichlet problem for the Laplace equation divide the domain into at most n parts. Carry over Courant's theorem to the case of systems (when the zeros form a set of codimension greater than 1).

1983-3. Can one carry over the Conley–Zehnder theory to reversible systems (*the latter resemble Hamiltonian systems so much that one would like to treat the property of being Hamiltonian as a variety of "superreversibility"*)?

1983-4. Let N lines be given in the real plane, and their complement be chess-like painted black and white. What is the greatest difference between the number of black and white regions?

1983-5. How many points of maximum can a polynomial of degree d in two (n) variables have? In particular, what would it be if all $(d-1)^2$ critical points are real?

1983-6. Find local contact classification of pairs of surfaces in $J^1(\mathbb{R},\mathbb{R})$ (in C^∞).

1983-7. How many nondegenerate periodic orbits can a diffeomorphism of $\mathbb{S}^1$ have if it is given by a trigonometric polynomial of degree n? The same question for a smooth map which is onto, or for a diffeomorphism which is given trigonometrically-rationally.

1983-8. Investigate real forms of reflection groups.

1983-9. Is it true that the number of periodic trajectories of a diffeomorphism of $\mathbb{S}^1$ is bounded by the integers which are the invariants (e. g., genera, bidegrees) of the algebraic self-correspondence that is the complexification of the diffeomorphism?

1983-10. Consider a projection $\mathbb{R}^{(n=)3} \to \mathbb{R}^{(k=)2}$ and the preimage of the integral points $\mathbb{Z}^{(k=)2}$, that are parallel lines (subspaces of dimension $n-k$) in $\mathbb{R}^{(n=)3}$. Consider a generic curve (manifold of dimension $k-1$) γ in $\mathbb{R}^{(n=)3}$ and its linking number with all the parallel lines (subspaces of dimension $n-k$). Investigate the behavior of the linking number under dilations of γ in terms of inflection points of γ. (If $n=k$ then this is a question about the number of integral points in a domain!)

1983-11. Is it true that the integrals $I(h) = \oint_{H=h}(P\,dx + Q\,dy)$ with varying polynomials P, Q form a Chebyshev system (or, at worst, the number of zeros is not too much greater)? Here, for instance, H is a cubic polynomial y^2+x^3-x. A similar question is also interesting about perturbations of other integrable polynomial systems of the Lotka–Volterra type [where $H = x^{\alpha}y^{\beta}z^{\gamma}$, $z = 1-x-y$, with the corresponding (non-polynomial) P, Q].

1983-12. Carry over the relation of indeterminacies (which connects projections of a Lagrangian manifold onto p- and q-subspaces) to Lagrangian manifolds with singularities and to the duality of convex polytopes. *For example, the stronger is a singularity of an oscillatory integral* (*as the wave length* $h \to 0$)*, the less is the number of points* (*in the* λ*-space*) *with this asymptotic* (*since* $S(x,\lambda) - \lambda x$ *is a Morse function in* (x,λ))*. But one can probably say more*!

1983-13. De Rham mixed structure theory: Define filtrations in a neighborhood of a singularity of a form in the *real* case in terms of the type of the singularity.

1983-14. Describe the Gibbs distribution of the density evolution under the action of a small diffusion ε and a flow v with multivalued potential U on a non-simply connected manifold as $\varepsilon \to 0$: $u_t + (uv)_x = \varepsilon\Delta u$, $v = -\operatorname{grad} U$ (*e. g., v is a pseudoperiodic function on* $\mathbb{R}^2$*, $v = ax + by +$ periodic part, and u is a function on a torus* $\mathbb{R}^2/$*periods*). Describe how the limit is being approached (for a generic U).

1983-15. Is it true that the singularities of the ellipticity and hyperbolicity boundaries in generic families are the same (topologically, smoothly) as the singularities of graphs of functions $\max_y F(x,y)$ for generic families F?

1983-16. Is it true that the number of limit cycles emerging from a singular point of an analytic system, is bounded (except for systems forming a set of codimension infinity, or possibly except for the *integrable* ones only)?