1982

1982-1. Is the symplectic structure of a neighborhood of the Lagrangian opening of the swallowtail standard?

1982-2. The Morse–Darboux super-lemma.

1982-3. Give a description of the liftable diffeomorphisms and fields in terms of their behavior on the singularities in the base.

1982-4. In the theory of integrable systems, Coxeter groups A, D, E appear (A. M. Perelomov and others). In the theory of integrable systems with a boundary (E. K. Sklyanin), do $H_{3,4}$ also appear?

1982-5. Describe the shapes of the resonance zones for torus mappings defined by trigonometric polynomials which perturb a translation (Mathieu type systems).

1982-6. Study the asymptotics of solutions of the thermoconductivity equation on differential forms with transfer ("dynamo"): uniqueness of the stationary solution in a given homology class.

1982-7. Investigate the singularities of the boundaries of the manifolds of elliptic and hyperbolic polynomials.

1982-8. It is known that the first sheet of a hyperbolic surface is convex. What can be said in this vein about the second, third etc. sheet?

1982-9. What happens to Legendre transforms (fronts) if the initial functions (hypersurfaces) depend on parameters and become singular for some parameter values? What perestroikas of dual objects take place there?

1982-10. Supplement the formal analysis of normal forms performed in the paper ARNOLD V. I. Reconstructions of singularities of potential flows in a collision-free medium and caustic metamorphoses in three-dimensional space. *Trudy Semin. Petrovskogo*, 1982, **8**, 21–57 (in Russian); *the English translation*: *J. Sov. Math.*, 1986, **32**(3), 229–257, by a study of smooth and analytic normal forms.

1982-11. Prove that taking gravitation into account in a dust-like medium does not affect the topological features of caustic perestroikas (with typical initial potential flow).

1982-12. Given a stratum $\mu = \text{const}$, what maximal value of μ do the adherent singularities have? For instance, the adjacency $P_8 \to E_6$ exists whereas $P_8 \to A_7$ and $P_8 \to D_7$ do not.

1982-13. Find normal forms for a typical contact structure in a neighborhood of the swallowtail (and investigate the hierarchies arising from the constraints on the ranks along submanifolds or on their tangent planes at the singularity).

1982-14. Develop the algebraic (analytic?) symplectic (contact) geometry that treats all the things in terms of ideals. Example: replace $df \neq 0$ with $\exists h$: the Poisson bracket of f and h is 1. Some theorems known in the nonsingular situation may happen to be more general (say, for isolated singularities?).

1982-15. Let

$$\prod_{k=1}^{3} \frac{z^N - z^{A_k}}{z^{A_k} - 1} = \sum_r p_r z^r$$

(A_k and N are natural numbers) be a polynomial with nonnegative coefficients p_r. Consider the number $B(a) = \sum(p_r : aN < r < (a+1)N)$.

Increase the fractions N/A_k (so that the coefficients of the polynomial remain nonnegative). Prove that the number $B(a)$ will then also increase (possibly nonstrictly).

In the n-dimensional case, A_k/N are the weights of a quasihomogeneous function with an isolated singularity at 0.

1982-16. Consider a Newton polyhedron Δ in $\mathbb{R}^n$ and the number $\mu(\Delta) = n!\,V - \sum(n-1)!\,V_i + \sum(n-2)!\,V_{ij} - \cdots$, where V is the volume under Δ, V_i is the volume under Δ on the hyperplane $x_i = 0$, V_{ij} is the volume under Δ on the hyperplane $x_i = x_j = 0$, and so on.

Then $\mu(\Delta)$ grows (non strictly monotonically) as Δ grows (whenever Δ remains coconvex *and integer*?). *There is no elementary proof even for $n = 2$.*

1982-17. Consider the boundary value problem $\Delta u = 0$ in the domain bounded by a quadric (say, a hyperbola in the plane, with the boundary value 1 on one component and 0 on the other). Then there exists a "natural" solution (moreover, there is a natural condition at the infinity which selects it).

Does there exist any reasonable filtration for harmonic functions and forms in the case of generic (hyperbolic?) algebraic hypersurfaces that yields a one-to-one correspondence between (relative) homology classes and harmonic representatives (for quadrics, the answers of Vainshtein and Shapiro would appear)? Is there a real version of the mixed Hodge structure?

1982-18. Develop the singularity theory for mappings between symplectic (contact) manifolds (a singularity is a violation of symplecticity).

1982-19. Explore symplectic correspondences, i. e., multivalued symplectomorphisms

$$X^{2n} \subset (A^{2n} \times B^{2n}), \quad (\pi_A^* \omega_A + \pi_B^* \omega_B)|_{X^{2n}} \text{ is symplectic.}$$

Find the hierarchy of the germs of such correspondences.

1982-20. Study the rationality of Poincaré series in natural analytic classification problems, e. g., for the germs of typical mappings in the worst dimensions where functional moduli are inevitable (virtually excluding the germs from a set of infinite codimension). Another example: apply this to the classification of the equations $y'' = F(x, y, y')$.

1982-21. What happens to the Givental triads when the quadraticity condition is violated (generically)?

1982-22. Can a divergence-free vector field tangent to the layers of Rieb's foliation have an exponential repulsion of trajectories?

1982-23. Investigate the singularities in the problem of bypassing an obstacle when the latter is not a hypersurface in the ambient space (e. g., for curves in $\mathbb{R}^3$).

1982-24. Can the center of mass of a convex domain in a homogeneous sphere coincide with the center of the sphere? Since it cannot, it makes sense to try to prove the existence of two closed curves (magnetic trajectories) of constant positive geodesic curvature on the sphere as follows. Fiber the space of convex disks over S^2 by associating to a disk its "center" point on the sphere. Find constrained critical points along fibers using variational methods, and then apply Morse theory techniques to look for critical points along the base.