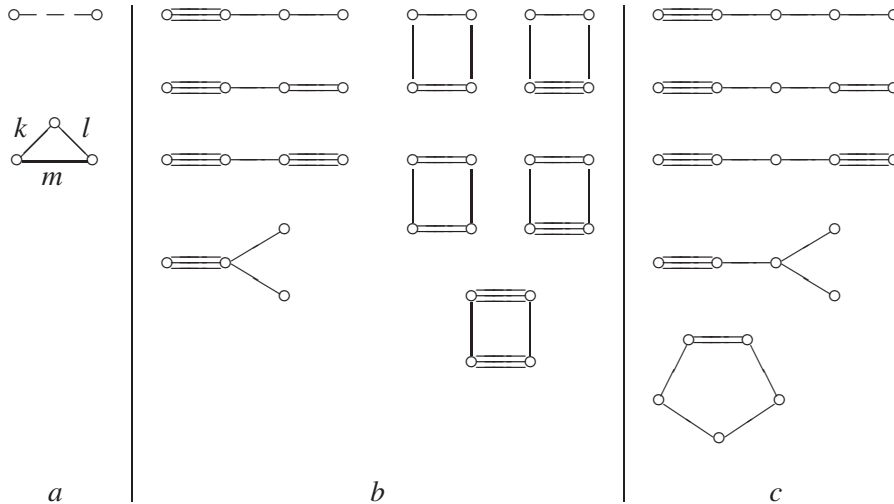


# 1981

**1981-1.** Lannér schemes (Coxeter schemes for the groups generated by reflections in the walls of simplices in the Lobachevskian space).



Simplices in the Lobachevskian space: a) the series on the plane, b) 9 simplices in the three-dimensional space, c) 5 simplices in the four-dimensional space.

Find applications of these schemes in singularity theory.

**1981-2.** Calculate the worst Nekhoroshev indices for generic Hamiltonians with  $n$  degrees of freedom (or at least the asymptotics of these indices for  $n$  large).

**1981-3.** Let

$$I_h(\lambda) = \int_{x \in \mathbb{R}^k} e^{iS(x,\lambda)/h} a(x,\lambda) dx,$$

where  $S$  is a generic function. Prove that there is the following bound for  $\lambda$  such that  $S(\cdot, \lambda)$  is a Morse function:

$$|I_h(\lambda)| \leq Ch^{k/2} \sum_{x \in \text{crit} S(\cdot, \lambda) \cap \text{supp} a(\cdot, \lambda)} \left| \det \frac{\partial^2 S}{\partial x^2} \right|^{-1/2},$$

where  $\text{crit}$  is the set of critical points of a function, and  $\text{supp}$  is its support. *Y. Colin de Verdière proved this for simple or parabolic singularities.*

**1981-4.** Does there exist an exact Lagrangian embedding of  $\mathbb{T}^2$  into the standard symplectic space  $\mathbb{R}^4$ ?

**1981-5.** Will a nonstandard contact structure of  $\mathbb{R}^3$  remain nonstandard after an arbitrary complexification?

**1981-6.** Evaluate the cohomology rings  $L_k = \lim_{n \rightarrow \infty} \pi_{n+k} T\lambda_n$ , where  $\lambda_n$  are the tautological Grassmann bundles over  $U(n)/O(n)$  or over  $U(n)/SO(n)$ , and  $T$  is the Thom space.

**1981-7.** A quasifunction is an exact Lagrangian embedded submanifold of  $T^*V$  that is isotopic to the zero section in the class of such embeddings. Critical points are intersections with the zero section. Conjecture: the number of critical points for a quasifunction is not less than for a function.

**1981-8.** What function on the collar can be extended over the ball without critical points?

**1981-9.** Consider closed contractible (bounding a disk on the universal covering) curves of constant geodesic curvature  $K \neq 0$  on a surface  $M^2$ . There are at least as many such curves as critical points of a function on  $M^2$ . *Counterexample: horocycles on a surface of constant negative curvature. However, for  $\mathbb{T}^2$  and  $S^2$  this conjecture has not been disproved.*

**1981-10.** Construct a bifurcation theory for optical caustics, in particular, prove that “flying saucers” caustics do not exist.

**1981-11.** Find a Lagrangian singularity related to the hypericosahedron group  $H_4$ .

**1981-12.** Find the (Zariski) relations between the (Zariski) relations of swallow-tails (and, in general, explore “syzygies,” or “noncommutative resolvents” of the fundamental groups of complements of algebraic hypersurfaces, associated with the sequence of complete flags of generic projections, and with generators and relations of the sequence of the fundamental groups of complements of discriminants of those projections).

**1981-13.** Evaluate the fundamental group of the space of nondegenerate plane curves of fixed degree  $d$ .

**1981-14.** Investigate the singularities of the density of a gravitationally evolving dust-like medium, if the initial potential field of velocities is generic (even on the line!).

**1981-15.** Can the barycenter of a convex part of a closed convex surface coincide with the barycenter of the surface?

**1981-16.** Is it true that a polynomial vector field on the plane has only finitely many limit cycles? *H. Dulac committed an error proving it.*

**1981-17.** Investigate the winding number of an analytic diffeomorphism of  $\mathbb{S}^1$  ( $x \mapsto x + a + b \sin x$ , etc.) as the limit (as  $\text{Im } a \rightarrow 0$ ) of the modules of elliptic curves formed by the orbit space for  $\text{Im } a \neq 0$ . What are the singularities of the analytic extension of the rotation number as a function of  $a$ ?

**1981-18.** Is there a kinematic magnetic dynamo in the topology of the three-dimensional ball  $B^3$ ?

**1981-19.** Give a contact version of the problem of bypassing an obstacle.

**1981-20.** Is it true that the singularities of the increment of a generic family of matrices (polynomials) are topologically equivalent to convex polyhedral or at least Morse functions (possibly, polyhedrally convex, Morse modified along the parameters, on which everything depends smoothly)?

**1981-21.** Explore singularities in typical controlled systems.

**1981-22.** Develop the theory of versal unfoldings of differential forms  $f(x)(dx)^\alpha$ .

**1981-23.** Evaluate the number of different “inflections” of algebraic surfaces of degree  $d$  in  $\mathbb{C}P^3$ .

**1981-24.** Investigate what the mixed structures and the spectra can provide for the Bruce problem about the maximum number of Morse points on a hypersurface of degree  $d$ .

**1981-25.** Work out a monodromy theory (of a representation of  $\pi_1$  of the complement to a bifurcation diagram) for complete intersections (not only for hypersurfaces): one should consider flags of hypersurfaces and sequences of Dynkin diagrams.

**1981-26.** Explore the effects of singularities (inflections of various types) on the asymptotics of the numbers of integer points on submanifolds of the Euclidean space and inside its domains (as well as the effects on the Diophantine approximations).

**1981-27.** Construct a theory of self-intersections of Lagrangian and Legendrian manifolds. To what extent are the self-intersections topologically inevitable (locally and globally)?

**1981-28.** Investigate the singularities of the convex hulls of  $M^3$  in  $\mathbb{R}^4$  (especially their modules).

**1981-29.** Elliptic coordinates in  $\mathbb{R}^n$ :

- a) a “magnet” generalization of the Ivory theorem (to forms);
- b) infinite-dimensional versions (with either discrete or continuous spectrum): what happens to the Jacobi formulae? in particular, to the surprising duality between the expression of impulses in the elliptic coordinates and the inversion formula of the coordinates;
- c) elliptic coordinates and the Hilbert transformation;
- d) equations of mathematical physics, integrable with the help of b).