1979

1979-1. How can one construct the quivers A, D, E from the singularities A, D, E (and their local rings)?

1979-2. Show that, for a generic function F, the function $\min_y F(x,y)$ is a topologically Morse function.

1979-3. Prove the semicontinuity of the spectrum of a singularity. Is it the spectrum of an oscillating system with μ degrees of freedom? In this case its interlacing by the spectrum of a close system with $\mu - 1$ degrees of freedom would follow from the Rayleigh–Courant–Fisher theory.

1979-4. Construct a "complexification" of the homology theory (replacing a boundary with a two-sheet branched covering). What is the complexification of orientation? (Apparently, it assigns an element of $\mathbb{Z} = \pi_1(\mathrm{U}(n))$ to a loop?)

1979-5. Construct the characteristic classes of Lagrangian singularities from the stable cohomology ring of complements of caustics.

1979-6. Do the real and the complex modality for a critical point of a function always coincide?

1979-7. Analyze the theory of envelopes in the framework of the theory of singularities. Find versal unfoldings, bifurcation diagrams, and the connection with symplectic and contact geometry.

1979-8. Why are caustics irreducible? How many irreducible components does the singularity manifold of a caustic have?

1979-9. Investigate the properties of the discriminants of non-quasihomogeneous Legendrian singularities. No topological classification has been found, even in the cases where a smooth classification (with moduli) is available.

1979-10. Describe the mixed Hodge structures of superpositions of functions.

1979-11. Investigate typical singularities of the boundary of the time-like attainability domain.

1979-12. Investigate the singularities of the time of shortest bypass of an obstacle.

1979-13. Is it true that the singularities of the boundary of the attainability domain in a generic controlled system are the same as those of a generic projection of a manifold with boundary? More generally, a "parameter" in optimization problems is a choice of control from a function space (which can have a boundary or other singularities). Are the singularities of the boundary of attainability in this case the same as those of a generic projection of finite-dimensional boundary manifolds with the same singularities?

1979-14. Is it true that the function of the shortest time within the attainable set has the same type of singularities as the minimum $\min_y F(x,y)$ of a generic family of functions?

1979-15. Investigate the bifurcations of the phase portrait in two-parameter generic systems of vector fields in the plane for the fields which are tangent to: a) a line, b) a pair of intersecting lines. (The normal forms for the eigenvalues are $0, \pm i\omega$ and $\pm i\omega_1, \pm i\omega_2$.)

1979-16. Study the number of zeros of the integral $I(h) = \oint_{\gamma_h} (P\,dx + Q\,dy)$, where γ_h is a closed curve from the (continuous) family of periodic orbits of a polynomial vector field [e. g., $\gamma_h = \{x, y : H(x,y) = h\}$, say, for $H = y^2 + x^3 - x$]—an infinitesimal version of the Hilbert 16th problem on cycles. What can be the maximal number of roots of $I(h)$ when $I(h)$ is not identically zero?

1979-17. Give an asymptotically sharp bound for the number of connected components of the space of nonsingular real algebraic hypersurfaces of degree d.

1979-18. Is the equality in the Petrovskiĭ–Oleĭnik inequality attainable?

1979-19. Does the Ragsdale conjecture hold? One may reformulate this conjecture as follows. Let $f(x,y,z)$ be a homogeneous polynomial of an even degree, $F_\pm = f \pm t^2$ and $\mathbb{R}V_\pm$ is the local level surface $F_\pm = \pm\varepsilon$. The Ragsdale conjecture is in the estimates of the number of components

$$b_0(\mathbb{R}V_+) \le h_1^{2,2}(F_+), \quad b_0(\mathbb{R}V_-) \le h_1^{2,2}(F_-) + 1$$

in terms of the mixed Hodge structure (if f has appropriate sign).

1979-20. Give the best possible estimates (through the degree or a Hodge number) for the individual Betti numbers of real algebraic hypersurfaces, in particular, for the number of components b_0. Probably, it is easier to estimate the numbers b_0, $b_0 - b_1$, $b_0 - b_1 + b_2$, ..., etc. and the combinations of the local type Morse numbers M_0, $M_0 - M_1$, $M_0 - M_1 + M_2$, ... (M_i is the number of critical points of index i merging at zero for some Morsification of the homogeneous equation of a hypersurface).

1979-21. What is the maximal number of handles that a component of an algebraic surface of degree n in $\mathbb{R}P^3$ can have?

1979-22. Estimate the number of ovals of a curve with a fewnomial equation, through the number of its terms.

1979-23. How many nonconvex ovals can a plane algebraic curve of degree n have?

1979-24. Does the isotopy type of the pair (plane M-curve, its complex orientation) determine a connected component in the space of nonsingular projective real curves of a given degree?

1979-25. Explore the fundamental group π_1 of the complement of the set of singular hypersurfaces in the complex projective space of all hypersurfaces of a fixed degree in $\mathbb{C}P^m$. Find the corresponding monodromy group (a representation of π_1 by automorphisms of the homology group of a hypersurface).

1979-26. Consider the system $\dot{x} = P(x,y)$, $\dot{y} = Q(x,y)$ where P, Q are polynomials of the second degree, and let $H(x,y)$ be a first integral of this system (not necessarily a polynomial one). How many limit cycles can emerge from components of level curves of H by small variations of P, Q leaving them quadratic polynomials?

1979-27. In the system of differential equations $\dot{x} = P(x,y)$, $\dot{y} = Q(x,y)$, let P, Q be power series starting with homogeneous polynomials P_n, Q_n of degree n. Is it true then that for almost all pairs (P_n, Q_n) the number of limit cycles emerging from the origin by a small perturbation of the system, is bounded by a constant depending only on n?