1978

1978-1. Investigate the topological properties of functions $f(x) = \max_y F(x,y)$.

1978-2. Explore the singularities of the boundary of the attainability manifold in a typical controlled system.

1978-3. Explore the singularities of the Nekhoroshev steepness indices (the stratification of the variety of Hamilton functions with respect to the indices). Calculate the indices of a typical system with 1, 2, 3 degrees of freedom at all the points.

1978-4. Let $\{I_\alpha\}$ be a collection of first integrals of a Hamiltonian system. Assume that $\{I_\alpha\}$ is closed under taking Poisson brackets, so that $(I_\alpha, I_\beta) = \mathcal{F}_{\alpha\beta}(I)$. Is it possible to replace $\{I_\alpha\}$ with $\{J_\alpha\}$ such that $(J_\alpha, J_\beta) = \sum_\gamma C^\gamma_{\alpha\beta} J_\gamma$? (If not, what deformations of the initial terms Lie algebra are not equivalent to each other?)

1978-5. Formalize the principle: whatever is good, is also delicate.

1978-6. Relaxed Hilbert 16th problem.

1978-7. What resonances in a three-frequency Hamiltonian system are strong (the strong resonances in a two-frequency system are $|\omega_1| : |\omega_2| = 1:1, 1:2, 1:3, 2:3, 1:4, 3:4, 2:5, 4:5$)?

1978-8. Describe the boundary singularities B_μ and C_μ that appear in the problem of bypassing an obstacle.

1978-9. How many cycles emerge from generic two-parameter bifurcations when eigenvalues pass through $\pm i\omega_1$, $\pm i\omega_2$ (in the corresponding slow system, i. e., for a vector field in the plane that is tangent to the sides of an angle, from a bifurcation with nonzero eigenvalues) in a two-parameter family of such fields, or—which is the same—for $(\mathbb{Z}_2 \times \mathbb{Z}_2)$-equivariant fields in the plane?

1978-10. Investigate the stratification of the manifold of linear elements of generic surfaces near each of the 10 strata.

1978-11 (V. L. Popov)**.** Find a connection between the theory of singularities and the quotients of $\mathbb{C}^2$ by finite subgroups of $\mathrm{U}(2)$ (not $\mathrm{SU}(2)$!).

1978-12. Investigate geometric (topological?) properties of p-real submanifolds in $\mathbb{C}^N$ or in a Kähler manifold (with a restriction on the dimensions of the intersections of the tangent planes with the tangent planes multiplied by i).

1978-13. Investigate the relations between Smith's theory of complex conjugations and the mixed Hodge structure on a manifold.

1978-14. Investigate Lie subsemigroups (e. g., of $\mathrm{SL}(2,\mathbb{R})$) and their tangent cones at 1.

1978-15. How many limit cycles can emerge from a zero of a Γ-nondegenerate vector field with a given Newton diagram Γ? Is it true that their number is less than some constant $N(\Gamma)$?

1978-16. Investigate the singularities of Gaussian maps globally.

1978-17. Investigate the theory of symmetric hyperbolic systems of partial differential equations in the framework of singularities.

1978-18. Construct explicitly the local topological classification of Lagrangian and Legendrian maps in the cases where the smooth classification has modules, or even functional modules. *The smooth classification has been described by V. M. Zakalyukin* (*it contains functional parameters*) *up to dimension* 10*, inclusive of the mapped Lagrangian or Legendrian manifold.*

But the following is not clear:

a) *Does the Zakalyukin class define the topological type of the Lagrangian* (*Legendrian*) *map*? *That is, is this type constant along each class*?

b) *Does this class define the topology of the decomposition into simpler classes of a neighborhood in the space of jets*? *That is, are the bifurcation diagrams locally diffeomorphic or at least homeomorphic*?

c) *Here the bifurcation diagram can be interpreted as*:

— *A*: *discriminant* (*a bifurcation diagram of zeros*);

— *B*: *bifurcation diagram of functions* (*in the truncated base*);

— *C*: *the projection of A onto B*;

— *D*: *the decomposition into Lagrange classes in the space of jets*;

— *E*: *the decomposition into Legendre classes.*

d) *Similar questions for multi-jets.*

e) *In order to apply transversality arguments to these stratifications of universal objects we need to know whether Whitney's A and B conditions are satisfied.* (*Conjecturally no, hence Zakalyukin's "stratification" is subject to a refinement*!).

1978-19. Investigate the bifurcations of type D_5 in the 3-space topologically (*the problem has been studied by V. I. Bakhtin*).

1978-20. Investigate the singularities of bicaustics of type D_5, up to diffeomorphisms.

1978-21. Investigate the process of sweeping the bicaustic D_4, up to equivalence (strong equivalence): given three smooth curves $\varphi_i : (\mathbb{R},0) \to (\mathbb{R}^2,0)$ starting from 0 with the same velocity $v \neq 0$, in all other generic. The equivalence is provided by the diagrams

$$\begin{array}{ccc} (\mathbb{R},0) & \xrightarrow{\varphi_i} & (\mathbb{R}^2,0) \\ \big\downarrow{\scriptstyle\tau} & & \big\downarrow{\scriptstyle h} \\ (\mathbb{R},0) & \xrightarrow{\psi_i} & (\mathbb{R}^2,0) \end{array}$$

(where the diffeomorphisms τ and h are independent of i); the strong equivalence is: $\tau(t) = t + \text{const}$.

1978-22. What is the behavior of the mixed Hodge structure of a singularity under the action of a complete monodromy group? (This can distinguish subgroups in π_1?)