1977

1977-1. Investigate the connection between the spectral sequence of the Newton filtration and the mixed Hodge structure of a Γ-nondegenerate singularity.

1977-2. Deduce generalizations of Petrovskiĭ's inequalities for curves with singularities from the mixed Hodge structures (hypersurfaces, etc.).

1977-3. Give a classification of unimodal boundary singularities.

1977-4. Give a classification of the simple singularities in the presence of a fixed singular hypersuface (or another algebraic subvariety).

1977-5. Explore the discriminant of H_3.

1977-6. Axiomatize the theory of complete and linearized invariants convolutions.

1977-7. Determine and investigate the indices of singular points of 1-forms on singular varieties.

1977-8. How do the Reidemeister and Ray–Singer torsion appear in singularity theory?

1977-9. Give a classification of nondegenerate quasihomogeneous maps $\mathbb{C}^2 \to \mathbb{C}^3$ and $\mathbb{C}^3 \to \mathbb{C}^3$ (similar to the decomposition of the space of maps $\mathbb{C}^2 \to \mathbb{C}^3$ into three types, and of the space of maps $\mathbb{C}^3 \to \mathbb{C}^3$ into seven types).

1977-10. Prove Lyashko's statements about the Poincaré polynomial of a quasihomogeneous map $f : \mathbb{C}^m \to \mathbb{C}^n$ with weights A_i in the domain and D_i in the range:

$$p(t) = (-1)^{m-n}\, t^{\Sigma D - \Sigma A} \left[-1 + \operatorname*{res}_{s=0} \prod \frac{1 - st^{A_i}}{s - st^{A_i}} \prod \frac{s - st^{D_i}}{1 - st^{D_i}} \frac{s\,ds}{1-s} \right] + \\ + \frac{\prod\left(1 - t^{D_i}\right)}{\prod\left(1 - t^{A_i}\right)} \left[\sum t^{-D_i} - \sum t^{-A_i} + 1\right].$$

For $m-n=1$,

$$p(t)=\frac{\prod(1-t^{D_i})}{\prod(1-t^{A_i})}\left[\sum t^{-D_i}-\sum t^{-A_i}+1\right]+t^{\Sigma D-\Sigma A}.$$

For $m-n=2$,

$$p(t)=\frac{\prod(1-t^{D_i})}{\prod(1-t^{A_i})}\left[\sum t^{-D_i}-\sum t^{-A_i}+1+t^{\Sigma D_i-\Sigma A_i}\right]-t^{\Sigma D-\Sigma A}.$$

If $m-n=1$, then $p(t)=t^{\Sigma D-\Sigma A}h(t)$, where h is the Poincaré polynomial of the Hamm–Gruel filtration:

$$h(t)=(-1)^{m-n}\left[-1+\operatorname*{res}_{s=0}\prod\frac{st^{A_i}+1}{t^{A_i}-1}\prod\frac{st^{D_i}-s}{st^{D_i}+1}\frac{ds}{s(s+1)}\right],$$

$$\Omega_{\mathrm{rel}}=\Omega^{m-n}/d\Omega^{m-n-1}+df\wedge\Omega^{m-n-1}$$

(the slash here means "modulo").

For $m-n=2$ this fails: $D_1=D_2=2$, $A_1=\cdots=A_4=1$, $h(t)=3t^4+4t^3$, $p(t)=2t^{-2}+4t^{-1}+1$. If $m-n=2$, then $h(1)=\mu(1)$. It is not known whether this is the case if $m-n>2$.

1977-11. Investigate the mapping that associates with each (unordered) set of critical points (of a function from a versal deformation) the (unordered) set of critical values. Investigate also the corresponding maps: (ordered sets of critical points) $\rightsquigarrow$ (unordered sets of critical values). Find the discriminants, fundamental groups and other invariants of branched coverings. How do the critical points rearrange after a circuit around a caustic?

1977-12. Investigate the bifurcations (with the parameters $\mathrm{Re}\,\varepsilon$, $\mathrm{Im}\,\varepsilon$) of the family of vector fields on the plane $\dot{z}=\varepsilon z+Az|z|^2+\bar{z}^3$ if the values of A are generic. (Conjecturally, they are the topologically versal deformations of $\mathbb{Z}_4$-symmetric fields for each of the 48 domains in the A-plane.)