1975

1975-1. Every interesting discrete invariant of a generic singularity with Newton polyhedron Γ is an interesting function of the polyhedron. Study: the signature, the number of moduli, the singularity index, the integral monodromy, the variation, the Bernstein polynomial, and μ_i (for generic sections).

1975-2. Is it possible to reconstruct the Newton polyhedron Γ from a Γ-nondegenerate function $f \in \mathfrak{m}^3$? In the quasihomogeneous case, is it possible to reconstruct the exponents? Is the main term reconstructible (or are those on the faces)?

1975-3. Let f be a quasihomogeneous but degenerate function. Is it possible to make the Newton polyhedron of f smaller by a quasihomogeneous coordinate transform? This is a particular case of the question of whether any function with $\mu < \infty$ is stably equivalent to a Γ-nondegenerate one.

1975-4. Let a function f be Γ-nondegenerate. Is it true that there exists a correct upper basis $\{e_k\}$ such that $f \sim f_0 + \sum c_k e_k$? Does there exist a correct upper basis serving for all sums f_0 with upper summands? (If yes, then the answer to the first question is positive.)

1975-5. Let $(\alpha_1, 1)$ and $(\alpha_2, 1)$ be two types of quasihomogeneity with affinely equivalent patterns (i. e., sets of integers $m \geq 0$ of the hyperplane $\{m : (m, \alpha) = 1\}$). Is it true that the upper patterns $\{m > 0 : (m, \alpha) = 1 + \beta\}$ are mutually equivalent (with a non-monotone re-enumeration $\beta_1 \mapsto \beta_2$), and that the upper basis of the first singularity is mapped to the upper basis of the other one?

1975-6. The stratum $\mu = \text{const}$ of a quasihomogeneous function in the standard versal deformation is linear and generated by weakly upper monomials. Does this hold for a Γ-nondegenerate function? (Generally—is the stratum $\mu = \text{const}$ smooth?)

1975-7. Can the complex singularities belonging to distinct strata $\mu = \text{const}$ be topologically equivalent?

1975-8. Is the singularity index semicontinuous?

1975-9. Is it true that the number $s(\mu)$ of the strata $\mu = \text{const}$ with $\mu = 32$ is a power of 2? For $\mu = 1, 2, 4, 8, 16$ we have $s(\mu) = 1, 1, 2, 4, 32$, respectively. Is there a logical pattern in the sequence $s(\mu) = \mathbf{1}, \mathbf{1}, 1, \mathbf{2}, 2, 3, 3, \mathbf{4}, 4, 7, 11, 15, 14, 17, 22, \mathbf{32}$, where the boldface numbers are the values of $s(\mu)$ that correspond to $\mu = 1, 2, 4, 8, 16$?

1975-10. Is the set of non-equivalent quasihomogeneous patterns with a given number n of variables finite? The equivalence is the combinatorial (or affine?) type of the convex hull of the pattern $\{m \in \mathbb{Z}^n : m \geq 0, (m, \alpha) = 1\}$.

1975-11. Is it true that in the complex case the complement of the bifurcation diagram of a function is always a $K(\pi, 1)$ space? Are the components contractible in the real case? *Conjecturally* no*, although R. Thom had thought that* yes!

1975-12. Does every real-valued function have a real Morsification (with μ real critical points)?

1975-13. What is the minimal number of critical values obtained by a perturbation of a critical point of multiplicity μ with μ Morse critical points? Conjecturally it is $n+1$, where n is the number of variables (or corank).

1975-14. Is the corank a topological invariant?

1975-15. What singularities can absorb A_1? split A_1 off? Why is every stratum $\mu = \text{const}$ connected to the stratum A_1 by a chain of strata of all codimensions?

1975-16. Suppose $f \oplus g \sim f \oplus h$ (f, g, h are isolated singularities). Is it true then that $g \sim h$?

1975-17. Give an "objective" definition of a series of singularities.

1975-18. List all decompositions of simple singularities.

1975-19. Calculate the stable cohomology ring of the complement of bifurcation diagrams: a) of functions of n variables, b) stable over $n \to \infty$.

1975-20. Compose a list of simple singularities of maps from m-dimensional manifolds to n-dimensional ones.

How does the A–D–E classification show up in this list?

1975-21. Express the main numerical invariants of a typical singularity with a given Newton diagram (e. g., the signature, the genus of the 1-dimensional Milnor fiber) in terms of the diagram.

1975-22. The problem of stabilization of invariants: investigate the behavior of the main invariants of a singularity when adding squares of new variables.

1975-23. Compare the stratifications of real and complex singularities of functions. Distinguish M-singularities among real forms. Compare real and complex modalities. Is a complex stratification always the complexification of a real one?

1975-24. Investigate the stratum $\mu = \text{const}$ (defined by the condition that the codimension of the orbit is constant). Is the stratum smooth (for algebraic group actions, for natural problems of the singularity theory, e. g., for the classification of singularities of caustics and wave fronts)?

Is it true that every such stratum becomes irreducible in the base of the complex versal deformation of some suitable "deeper" singularity?

Does the cohomology ring of the complement of the stratum stabilize in this "growing" base?

1975-25. Investigate the Lagrangian singularities of bifurcating caustics from the cosmological "pancake theory" of Zeldovich (in particular, taking into account the gravitation and particle fusion, and for nonpotential flows).

1975-26. Evaluate the normal forms of versal deformations of matrices of various types (symmetric, unitary, etc.), and investigate the corresponding bifurcation diagrams and cohomology rings.

1975-27. Explore the asymptotics of oscillatory integrals (in particular, find uniform estimates near singularities of caustics and calculate the highest individual singularity indices appearing unremovable in typical families with a given number of parameters).

Carry over these estimates to integrals of the saddle-point method.

1975-28. Investigate the singularities of envelopes of typical families of submanifolds from the viewpoint of the symplectic and contact theory of Lagrangian and Legendrian maps.

1975-29. Explore the singularities of solutions of generic variational problems (as well as those appearing in typical families with prescribed or not prescribed finite number of parameters).

1975-30. Investigate the singularities of implicit differential equations (both ordinary and partial).