1974

1974-1. The reconstruction of a quasihomogeneous Lie algebra from its root system. Consider a collection of positive exponents (weights) of quasihomogeneity w_i of the coordinates x_i in $\mathbb{C}^n$ ($i = 1, \ldots, n$). A *generator* of a quasihomogeneous Lie algebra is a monomial $x^m \partial/\partial x_i$ of weight zero ($m \in \mathbb{Z}^n$, $m_j \geq 0$, $\sum w_j m_j = w_i$). A *root* of this generator is a vector $\widetilde{m} = m - 1_i \in \mathbb{Z}^{n-1} = \{\widetilde{m} : \sum w_j \widetilde{m}_j = 0\}$. Is it possible to reconstruct the Lie algebra generated by these generators (up to an isomorphism of Lie algebras) from the system of its roots, considered up to a linear transformation of the hyperplane $\mathbb{R}^{n-1}$ that does not necessarily preserve the coordinate hyperplanes $m_i = 0$ in $\mathbb{R}^{n-1}$?

The system of weights cannot be reconstructed, but the algebra is almost reconstructible (*modulo the signs of some structural constants*). *In all the examples ever considered, different choices of these signs result in isomorphic algebras. But it is unclear whether this is always the case.*

1974-2. In the theory of the duality of convex polyhedra there appears a Lagrangian or Legendrian manifold with singularities. In the same way, in optimal control theory there appear generalizations of Hamiltonian systems with nonsmooth Hamiltonians (a manifold of phase curves can pass through one point, as in the case of the Hamiltonian $H = |p_1| + |p_2|$). Nevertheless, their "flows" in some sense satisfy the Liouville theorem and should be considered as generalized symplectomorphisms (which are, most probably, not maps but Lagrangian submanifolds with singularities in the product space).

Develop a theory of Lagrangian manifolds with singularities, and generalized symplectomorphisms applying to such situations (and even obtain estimates from below for the number of intersection points of exact Lagrangian manifolds, and for the number of fixed points of exact symplectomorphisms, generalizing the Poincaré "geometric theorem").

1974-3. Find all singular values of the moduli of parabolic singularities (that change the topological or the combinatorial type of the projection of the manifold of the discriminant's singularities onto the bifurcation diagram of functions, i. e., the set of clauses for a decomposition of a critical point into several clusters of simpler critical points on (generally) several critical levels, realized by small deformations of the function). What are the elliptic curves corresponding to these values of moduli celebrated for?

1974-4. Find the classification problem of the theory of Lagrangian (Legendrian?) singularities, the answer to which would be in natural bijection with the list of Coxeter reflection groups.

1974-5. Find applications of the (Shephard–Todd) complex reflection groups to singularity theory.

1974-6. Symplectize the topology: Poincaré's index theory of singular points, apparently, turns into the theory of fixed points of symplectomorphisms and generalizations of Poincaré's last geometric theorem (i. e., to a generalization of the Morse theory). Do other topological theories have symplectizations? *Similarly to a noticeable difference between $\mathbb{Z}_2$ and its complexification $\mathbb{Z}$, the symplectization can also be as far from the initial object as the Coxeter group C_k is from A_k.*

1974-7. Classify the simple singularities of functions on a manifold with an action of a group (for example, finite) up to equivariant diffeomorphisms (commuting with the group action).

1974-8. Investigate the typical perestroikas of a wave front moving with time (and of the corresponding Legendrian map).

1974-9. Give a topological classification of the Legendrian singularities corresponding to the parabolic critical points of functions.

1974-10. A conic singularity over a given base carries topological invariants of the base into the singular point. For non-conic singularities (e. g., quasihomogeneous?) one may try to find traces of the discrete invariants of the base (e. g., the rank and the signature of the Milnor fiber?) in the local algebra of the singularity.

What algebraic objects are encountered in this way? What happens to the characteristic classes and numbers?