1973

1973-1. Describe the typical singularities appearing in the problems on differential games.

1973-2. Find the typical singularities of convex hulls.

1973-3 (S. Smale – J. Debreux)**.** Apply the singularity theory to economic models.

1973-4. Prove that the equilibria points stability problems and the problems about limit cycles are algorithmically unsolvable.

1973-5. Explore the normal forms of implicit differential equations unresolved with respect to derivatives, and their bifurcations.

1973-6. Investigate three-parameter bifurcations of the topological type of the dynamics in a neighborhood of a singular point of a vector field (the zero and an imaginary pair, etc.).

1973-7. The problem of smoothness of the stratum $\mu = \text{const}$.

1973-8. The problem of semicontinuity of the modality (the number of moduli).

1973-9. Investigate the lower deformations of the critical points of functions (a generalization of the theory of algebraic hypersurfaces!): the structure of discriminants, fundamental groups, vanishing cycles, etc.

1973-10. Prove the "$(2,2)$" formula for the number of moduli of a Γ-nondegenerate function in two variables, and deduce analogous "stereometric" formulae for the other invariants (μ, etc.).

1973-11. Generalize the classification of the admissible types of quasihomogeneity of nondegenerately-quasihomogeneous critical points (which is known only in the case of two and three variables). The question is related to the theory of cyclotomic polynomials.

1973-12. Is it true that the complement of the discriminant of a function's singularity of finite multiplicity is $K(\pi, 1)$?

1973-13. Investigate the topological invariants of bifurcation diagrams of functions (at least within the scope of tables, in order to work out general conjectures!) in the real and the complex case.

1973-14. What restrictions on the coexistence of singularities (on the same fiber, on different fibers) are imposed by the condition that the singularities belong to a versal unfolding of a given singularity of finite multiplicity (the problem is related to the 16th Hilbert problem)?

This is the problem that formed the basis of the semicontinuity of the spectrum of a singularity, estimates for the number of Morse points on a hypersurface, etc.

1973-15. Develop the theory of cobordisms of the critical points of functions.

1973-16. Carry over the achievements of the theory of critical points of functions to the study of smooth complex maps into spaces of greater dimension.

1973-17. Describe completely the stratification of the space of functions in two variables.

1973-18. Is there any relation between the Minakshisundaran–Pleijel coefficients and the coefficients of the polynomial whose value is the volume of the ε-neighborhood (e. g., for an isoperimetric embedding into $\mathbb{R}^N$)?

1973-19. Does each function have Morsifications with any number of critical values, from 1 to μ? How many distinct critical values are necessary in the real case?

1973-20. Find the transformation group preserving the ratio of the forms $\int u^2\,dx$ and $\int (u')^2\,dx$ in the space of functions u.

1973-21. Construct Dynkin diagrams for simple singularities as the quivers of some subspaces of local rings (derive the quivers from the structure of ideals?). *A. N. Shoshitaĭshvili suggested a construction that solves this problem for all cases except E_7, which is, therefore, unsatisfactory.*

1973-22. The Jacobian of the map $\binom{x}{y} \mapsto \binom{x}{xy}\binom{=u}{=v}$ is degenerate on the line $x = 0$, and the line $u = 0$ is not covered by this map (with the exception of the point 0). The Lyashko–Looijenga map for (unimodal) parabolic singularities has an analogous property. What is the general formulation of the corresponding conservation law: the more degeneracy in the domain, the more is uncovered in the range (or: the less is covered in the range, the more singularities are in the domain)?

1973-23. Is the asymptotic Hopf invariant (or helicity) of a divergence-free vector field in $\mathbb{S}^3$ invariant under volume-preserving homeomorphisms?

1973-24. Study the relation between the asymptotic Hopf invariant and the Reidemeister (*Ray–Singer*) torsion.

1973-25. A. D. Sakharov's conjecture: if a frozen-in vector field has linked or knotted trajectories, it cannot relax to arbitrarily small energies by the action of $\mathrm{SDiff}(B^3)$.

1973-26. The relaxation paradox: one cannot believe that formerly non-integrable fields have to relax to the eigenfields of the operator rot. What happens to them? Does the limit field encounter singularities? Or there is no limit field at all?

1973-27. Consider the mapping $\mathbb{C}^k \to \mathbb{C}^k$ associating to a point in a versal deformation the polynomial whose roots are the critical values of the corresponding function. For the versal deformation of the singularity A_k the multiplicity of this ramified covering, $(k+1)^{k-1}$, is equal to the number of trees with $k+1$ numbered vertices. Give a similar interpretation to the multiplicity of this mapping for other simple singularities (which is, according to O. V. Lyashko, $k!\,h^k/|W|$, where h is the Coxeter number, and $|W|$ is the order of the Weyl group of the singularity).

1973-28. Consider a random set of points in $\mathbb{R}^n$ with density ρ. Let $V(d)$ be the d-neighborhood of this set. Consider the averaged Betti numbers

$$\lim_{R\to\infty} \frac{b_i(V(d) \cap (\text{ball of radius } R))}{R^n} = \beta_i(d,\rho).$$

Investigate these functions.