1972

1972-1. Investigate the topology of the complement of the caustic $\Sigma^2_{\pm}$ in $\mathbb{C}^3$: is it true that this complement is a $K(\pi, 1)$ space?

1972-2. Investigate the monodromy group of the singularity $x^3 + y^3 + z^3$ (and also the topology of the complement of the discriminant).

1972-3. Is it true that $\min_y F(x, y)$ is topologically equivalent to a smooth function: a) for a generic F, b) always?

1972-4. Investigate the local convexity of the boundary of the stability domain (in the families of matrices and polynomials).

1972-5. Prove the uniform estimate for an oscillatory integral: how can one calculate the uniform index for a neighborhood in terms of the phase at the degenerate point?

1972-6. Is it true that the only singularities whose intersection form is positive or negative definite are A, D, E?

1972-7. Is the following conjecture on transversality of the stratification of a space of quadratic forms true: the manifold of quadratic forms in a Hilbert space that are determined by oscillations of arbitrary membranes is transversal to the stratified manifold of quadratic forms with multiple eigenvalues?

1972-8. Find "the most probable" representations of symmetry groups.

1972-9. Investigate the error of the method of averaging in the case of two frequencies, when in average the ratio of the frequencies changes with nonzero rate in the averaged motion (although the instantaneous rate of change in some fast phases changes its sign).

1972-10. Investigate the error of the method of averaging in generic multi-frequency systems under the assumption of passing through a resonance.

1972-11. Evaluate the cohomology of the braid groups of the series D and E.

1972-12. Classify the singularities of convex hulls of generic submanifolds in a vector space.

1972-13. Find the number of moduli for the Brieskorn singularities $\sum_i x_i^{a_i}$.

1972-14. Is it true that the complement of a bifurcation diagram is always a $K(\pi, 1)$ space?

1972-15. Prove that simple orbits coincide with orbits that are adherent only to orbits of smaller codimension (but not to unions of orbits of greater codimension).

1972-16. Find all the self-consistent gravitational potentials on the straight line (the stationary points, possibly generalized, of the Poisson–Vlasov equation).

1972-17. Prove that a diffeomorphism of the two-dimensional torus homotopical to the identity has at least four fixed points (counting multiplicities) and at least three of them are geometrically distinct, whenever this diffeomorphism preserves areas and leaves the center-of-mass invariant.

1972-18. Show that any orientation- and area-preserving diffeomorphism of the two-dimensional sphere onto itself has at least two geometrically distinct fixed points.

1972-19. Are the structurally stable maps of $\mathbb{S}^1$ into itself dense?

1972-20. Straightening the circle diffeomorphisms (by a smooth change of variables) for almost all the rotation numbers (*solved by M. R. Herman*) and the topological obstacle to analytic straightening: the existence of periodic orbits arbitrarily close to the real circle (*maybe, even in a neighborhood of any point of the circle*?). The similar obstacle to prolonging the reducibility annulus to a rotation by a holomorphic change of variables or the reducibility disk in Siegel's problem.

1972-21. The Floquet theory over the torus.

1972-22. A sufficiently curved submanifold is extremal in the Diophantine sense (with probability 1, the Diophantine exponent is the same as in the ambient space).

1972-23 (R. Thom)**.** A gradient vector field with a singular point has a trajectory entering the singular point with tangency to some straight line.

1972-24. Investigate the connections between the invariants of a singularity of a plane complex curve and the local fundamental group of its complement.

1972-25. Action of the monodromy M on the homology of the Milnor fiber. Decompose the singularity having included it in the family $f(z) - pz$ where $p \in \mathbb{C}^n$ is a parameter. Examine the bifurcation manifold $\Sigma = \{p, \varepsilon : \varepsilon$ is a critical value of the function $z \mapsto f(z) - pz\} \subset \mathbb{C}^{n+1}$. (This manifold is determined by the equation $\varepsilon = H(p)$, where H is the Legendre transform of f.) Consider $\pi_1(\mathbb{C}^{n+1} \setminus \Sigma)$ (germs at 0). Conjecturally, the properties of M (nilpotency, etc.) reflect the properties of π_1. For example, if a path $\varepsilon_0 e^{i\varphi}$, $0 \le \varphi \le 2\pi N$, commutes with all the generators of π_1, then is it true that it does not shift vanishing cycles (so that $M^N = 1$)?

1972-26. What are the restrictions imposed on the topology of a manifold by the hypothesis that the manifold is a degree n algebraic hypersurface in $\mathbb{R}^m$ (in $\mathbb{R}\mathrm{P}^m$)?

1972-27. Is it possible to represent an algebraic function $z(a,b,c)$, $z^7 + az^3 + bz^2 + cz + 1 = 0$, as one of the components of a superposition of algebraic functions in two variables? Find the conditions on the fundamental group, the adjacency of the strata, monodromy, and other topological invariants under which the algebraic function is not representable as a component of a superposition (conjecturally, these topological invariants are more complicated for the functions that are not representable in such a form).

In this problem algebraic functions can be replaced by "pseudoalgebraic" functions, which are topologically (or combinatorially) equivalent to them—conjecturally the nonrepresentability persists even for superpositions of such pseudoalgebraic maps.

1972-28. Find the three-dimensional characteristic class of the foliation of either $P(x,y) = C$ or $P\,dx + Q\,dy = 0$ in $\mathbb{C}\mathrm{P}^2 \setminus$ (singular points). (Here P and Q are polynomials.)

The following three problems are related to this class.

1972-29. Determine if this class is integral (for example, in the real case).

1972-30. Determine the conditions on the deformations of the coefficients or on the cobordisms that preserve this cocycle.

1972-31. Try to relate this class to limit cycles (not simply-connected fibers).

1972-32. Are the Boardman classes Σ^I *topologically* invariant?

1972-33. Prove that a symplectic diffeomorphism of a compact symplectic manifold M onto itself possesses at least as many fixed points as a smooth function on M has critical points, whenever this diffeomorphism is homologous to the identity.