1966

1966-1. What is the connection between the I-component $I(t)$ of the solution of the system

$$d\varphi/dt = \omega(I) + \varepsilon f(I,\varphi), \quad dI/dt = \varepsilon F(I,\varphi)$$

($\varphi \in \mathbb{T}^k$, $I \in \mathbb{R}^l$, $0 < \varepsilon \ll 1$) and the solution $J(t)$ of the "evolution equation"

$$dJ/dt = \varepsilon \bar{F}(J), \quad \bar{F}(J) := \frac{1}{(2\pi)^k} \oint_{\mathbb{T}^k} F(J,\varphi)\, d\varphi$$

with the same initial data on the interval $0 < t < 1/\varepsilon$?

1966-2. What is the behavior of orbits in the complement to the union of the invariant tori of a nearly integrable Hamiltonian system? Is it true, in particular, that these orbits exhibit no evolution in the s-th approximation, i. e., $|I(t) - J(t)| \ll 1$ for $0 < t < 1/\varepsilon^s$? Here I denotes the vector of the action variables, $J(t)$ is the solution of the s-th order "evolution equation" with the initial conditions $J(0) = I(0)$, while $0 < \varepsilon \ll 1$ is the perturbation parameter.

1966-3. Prove or disprove the following conjecture. Consider a nearly integrable Hamiltonian system with $k \geq 3$ degrees of freedom and with the Hamilton function $H_0(I) + \varepsilon H_1(I,\varphi)$, where (I,φ) are the action–angle variables. Then "generically" for every pair of neighborhoods of the tori $I = I'$, $I = I''$ with $H_0(I') = H_0(I'')$, there is an orbit passing through both neighborhoods provided that ε is sufficiently small.

1966-4. Let a diffeomorphism $A\colon q \mapsto q + f(q)$ of the torus $\mathbb{T}^2 = \{(q_1,q_2) \operatorname{modd} 2\pi\}$ preserve the measure $dq_1 \wedge dq_2$ and the center-of-mass:

$$\oint_{\mathbb{T}^2} f(q)\, dq_1 dq_2 = 0.$$

Prove that A has at least 4 fixed points counting multiplicities and at least 3 geometrically distinct fixed points.

1966-5. Let $\Omega = \mathbb{T}^k \times B^k$ ($\mathbb{T}^k = \{q \operatorname{modd} 2\pi\}$, $B^k = \{p \in \mathbb{R}^k,\ |p| \leq 1\}$) be the toroidal annulus equipped with the canonical structure $\omega^1 = p\, dq$, and let $A\colon \Omega \to \Omega$ be a canonical diffeomorphism homotopic to the identity transformation and such that each sphere $\{q\} \times \partial B^k$ is linked with its image on the covering of the boundary $\mathbb{T}^k \times \partial B^k$. Then A possesses at least 2^k fixed points counting multiplicities and at least $k + 1$ geometrically distinct fixed points.

1966-6. Investigate the ergodic properties of motions in the complement of the union of the invariant tori of a nearly integrable Hamiltonian system. In particular, is the entropy of such a system positive?