1965

1965-1. Let $A : \Omega \to \Omega$ be a globally canonical homeomorphism of the $2n$-dimensional toroidal annulus $\Omega = \mathbb{T}^n \times B^n$, where $\mathbb{T}^n = \mathbb{R}^n / 2\pi\mathbb{Z}^n$ denotes the n-torus while $B^n \subset \mathbb{R}^n$ is a domain in $\mathbb{R}^n$ homeomorphic to a closed n-dimensional ball. Let p_0 be an interior point of B^n, and $T \subset \Omega$ be the torus $\mathbb{T}^n \times \{p_0\}$. Do T and AT always intersect at not less than $n+1$ (geometrically distinct) points?

In this problem and the subsequent two problems, a mapping $A : \Omega \to \Omega$, where $\Omega = \mathbb{T}^n \times B^n$,

$$\mathbb{T}^n = \{q = (q_1, \dots, q_n) \operatorname{modd} 2\pi\}, \quad B^n \subset \mathbb{R}^n = \{p = (p_1, \dots, p_n)\},$$

is said to be *globally canonical* if it is homotopic to the identity transformation and

$$\oint_\gamma p\,dq = \oint_{A\gamma} p\,dq$$

($p\,dq = p_1\,dq_1 + \cdots + p_n\,dq_n$) for any closed curve $\gamma \subset \Omega$ (not necessarily homologous to zero).

1965-2. Let $A : \Omega \to \Omega$ be a globally canonical diffeomorphism of the $2n$-dimensional toroidal annulus $\Omega = \mathbb{T}^n \times B^n$, where $\mathbb{T}^n = \mathbb{R}^n / 2\pi\mathbb{Z}^n$ denotes the n-torus while $B^n \subset \mathbb{R}^n$ is a domain in $\mathbb{R}^n$ homeomorphic to a closed n-dimensional ball. Let p_0 be an interior point of the domain B^n and let $T \subset \Omega$, the torus $\mathbb{T}^n \times \{p_0\}$. Do T and AT always intersect at not less than 2^n points (counting multiplicities)?

1965-3. Let $A : \Omega \to \Omega$ be a globally canonical diffeomorphism of the $2n$-dimensional toroidal annulus $\Omega = B^n \times \mathbb{T}^n$, where $B^n \subset \mathbb{R}^n$ is a domain in $\mathbb{R}^n$ homeomorphic to a closed n-dimensional ball while $\mathbb{T}^n = \mathbb{R}^n / 2\pi\mathbb{Z}^n$ denotes the n-torus. Suppose that, for any $q \in \mathbb{T}^n$, the spheres $S^{n-1}(q) = \partial B^n \times \{q\}$ and $AS^{n-1}(q)$ are linked in $\partial B^n \times \mathbb{R}^n$ where $\mathbb{R}^n \to \mathbb{T}^n$ is the universal covering. Is it true that, in this set-up, the diffeomorphism A possesses at least 2^n fixed points in the annulus Ω (counting multiplicities)?